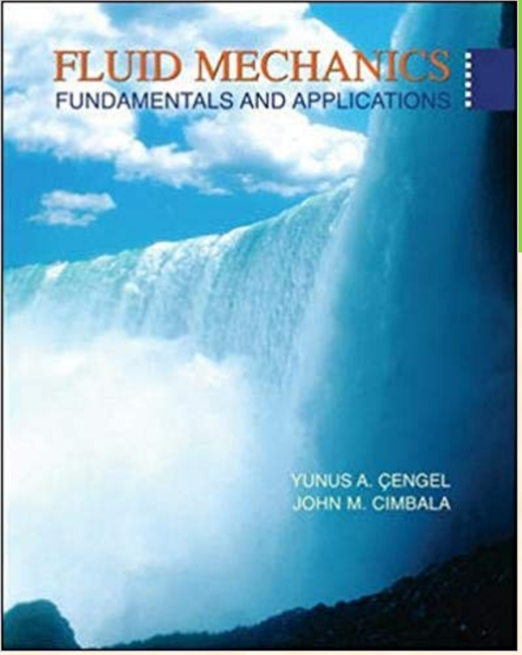

YUNUS A. ÇENGEL and JOHN M. CIMBALA, "Fluid Mechanics: Fundamentals and Applications", 1st ed., McGraw-Hill, 2006.

Course name

Fluid Mechanics I

Lecture-01- Chapter-05

Viscous effects and Flow Resistance

Lecture slides by

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Outline

- *Introduction*
- *Steady flow between parallel plate*
- *Laminar and Turbulent flow in pipes*
- *Entrance Region And Fully Developed*
- *Examples*

5-1: Introduction

The equation of motion for a real fluid can be developed from consideration of the force acting on a small element of the fluid including the shear stresses generated by the fluid motion and viscosity. The derivation of these equations called the Navier-Stokes equation.

- This **internal resistance** to flow is quantified by the fluid property *viscosity*, which is a measure of internal stickiness of the fluid.
- **Viscosity** is caused by cohesive forces between the molecules in liquids and by molecular collisions in gases. There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree.
- Flows in which the frictional effects are significant are called **viscous flows.** However, in many flows of practical interest, there are *regions* (typically regions not close to solid surfaces)
- where viscous forces are negligibly small compared to inertial or pressure forces. Neglecting the viscous terms in such **inviscid flow regions** greatly simplifies the analysis without much loss in accuracy. **See figure 1-15**

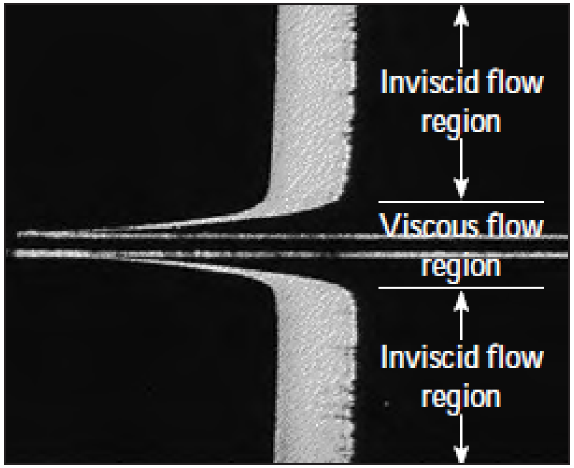


FIGURE 1–15
The flow of an originally uniform fluid stream over a flat plate, and the regions of viscous flow (next to the plate on both sides) and inviscid flow (away from the plate).

5-1: Introduction

- The viscosity of a fluid is a measure of its resistance to flow under an applied force.

- The greater the viscosity, the larger the force required to maintain the flow, and the more energy that is dissipated.

- Molasses has a high viscosity, water a smaller viscosity, and air a still smaller viscosity.

Figure 1 shows two flat plates separated by a thin fluid layer.

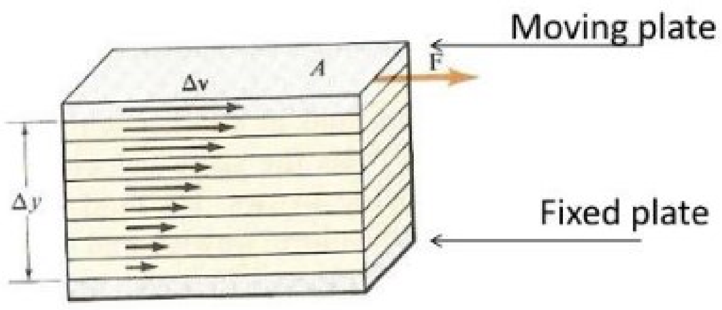


Figure 1

- The lower plate is held fixed, a force is required to move the upper plate at a constant speed.
- This force is needed to overcome the viscous forces due to the liquid and is greater for a highly viscous fluid

The force F is observed to be proportional to the area of the plates A and to the velocity of the upper plate Δv (the moving one) and inversely proportional to the plate separation Δy.

$$F = \eta A \frac{\Delta v}{\Delta y} \qquad (1)$$

Dimensions of Viscosity

$$[\eta] = \left[\frac{F/A}{\Delta v/\Delta y} \right]$$

$$[\eta] = \frac{MLT^{-2}/L^2}{LT^{-1}/L}$$

$$[\eta] = ML^{-1}T^{-1} = kg\, m^{-1} s^{-1} = Pa.s$$

M, L, and T stand for mass, length, and time respectively.

The S.I. Unit of viscosity is: 1 kg m^{-1} s^{-1} = 1 Pa s.

https://www.slideserve.com/iliana/14-p-341-viscous-fluid-flow

5-1: Introduction

From chapter 1

$\tau = \mu \frac{du}{dy}$

for three dimensional flow (Stokes law of viscosity)

$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$

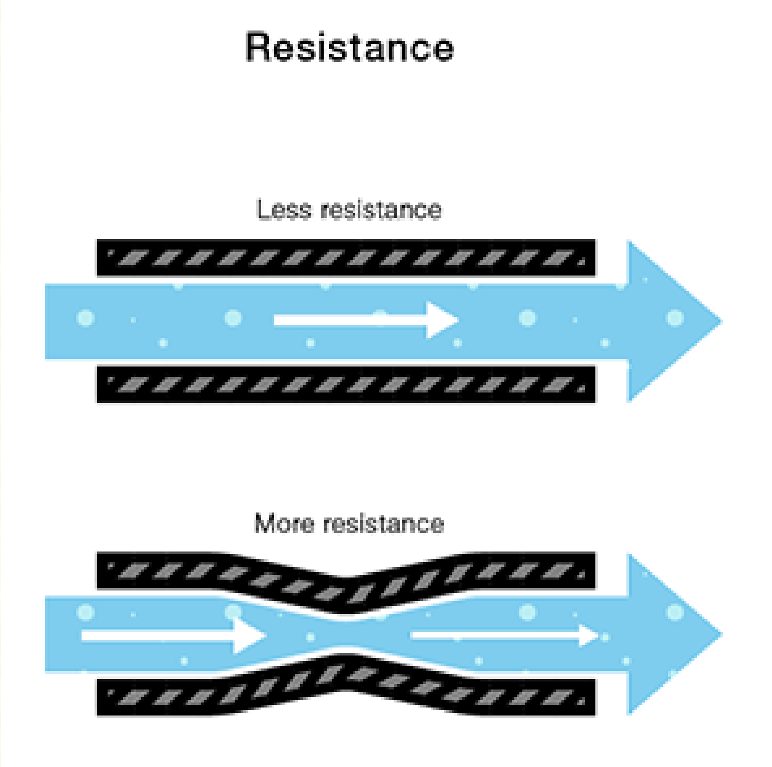
$\tau_{yz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial y}\right)$

$\tau_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$

where u, v, w the velocity of flow in the directions x, y, z for steady state one-dimensional respectively

i.e $v = 0$, $w = 0$ and $\frac{\partial u}{\partial z} = 0$

$\therefore \boxed{\tau = \mu \frac{du}{dy}}$ ——— *



Flow Resistance

- The flow resistance R_f is defined in general, as the ratio of the pressure drop to the flow rate: $R_f = \frac{\Delta P}{Q}$
- R_f defines the flow resistance whether the flow is laminar or not.

-The basic S.I .Unit of R_f eht si Pa.s.m^{-3}

But we use: kPa s m^{-3}

https://www.slideserve.com/iliana/14-p-341-viscous-fluid-flow

https://learn.sparkfun.com/tutorials/voltage-current-resistance-and-ohms-law/resistance

5-2 Laminar Incompressible: Steady flow between parallel plate

In equilibrium, **the net force acting on the plate** in the horizontal direction must be zero, and thus a force equal and opposite to *F* must be acting on the plate. This opposing force that develops at the plate–rubber interface due to friction is expressed as ***F*= τ *A***, where τ is the shear stress and *A* is the contact area between the upper plate and the rubber.

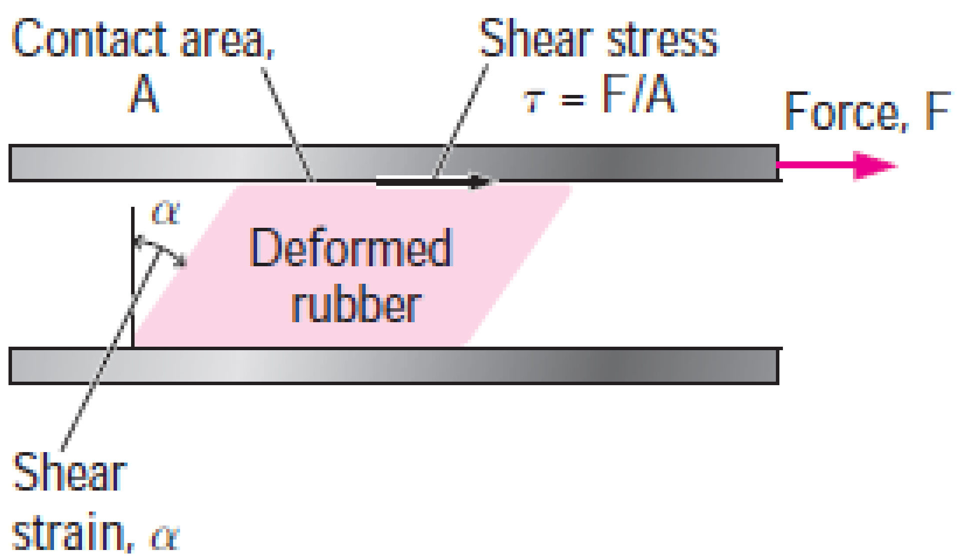
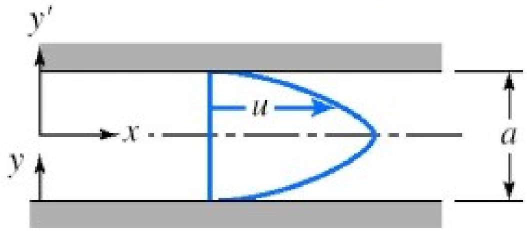


FIGURE 1–2
Deformation of a rubber eraser placed between two parallel plates under the influence of a shear force.

5-2 Laminar Incompressible: Steady flow between parallel plate

Fully Developed Laminar Flow Between Infinite Parallel Plates

✓ **Both Plates Stationary**



$$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

Fully Developed Laminar Flow Between Infinite Parallel Plates

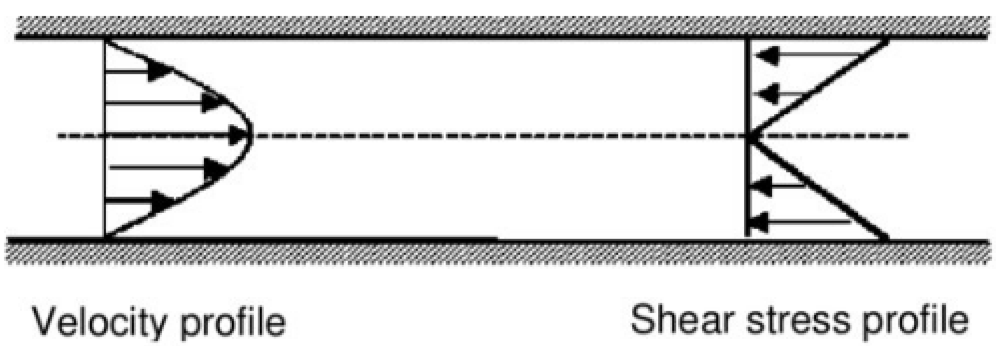
✓ **Both Plates Stationary**

- **Shear Stress Distribution**

$$\tau_{yx} = a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right]$$

- **Volume Flow Rate**

$$\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$$

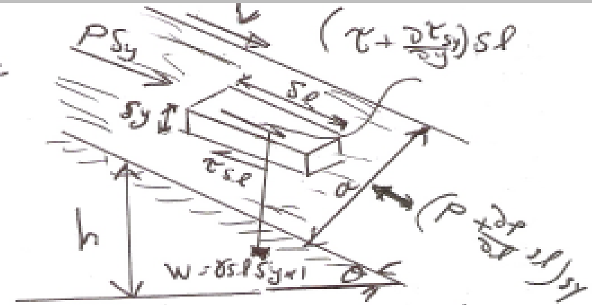


No slip boundary condition: velocity and shear stress profile between two wide, parallel plates in laminar flow

https://www.researchgate.net/publication/224603183_A_multi-function_microfluidic_module_for_mutation_detection/figures?lo=1

5-2 Laminar Incompressible: Steady flow between parallel plate

The upper plate moves parallel to the flow direction and there is a pressure variation in the direction as shown in the following Fig



-The layer of fluid moves of constant velocity u. and for steady flow - laminar flow, constant velocity. The equ of motion yields.

$$P\delta y - (P + \frac{\partial P}{\partial l}\delta l)\delta y - \tau\delta l + (\tau + \frac{\partial \tau}{\partial y}\delta y)\delta l + \gamma\,\delta l\,\delta y\sin\theta = 0$$

$$P\delta y - P\delta y - \frac{\partial P}{\partial l}\delta l\delta y - \tau\delta l + \tau\delta l + \frac{\partial \tau}{\partial y}\delta y\delta l + \gamma\delta y\delta l\sin\theta = 0$$

Dividing both side by $\delta l\delta y$ and subsituting that

$\sin\theta = -\frac{\partial h}{\partial l}$ we get

$$\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial l} + \gamma\frac{\partial h}{\partial l}$$

or $$\frac{\partial \tau}{\partial y} = \frac{\partial}{\partial l}(P + \gamma h)$$

Since $\tau = \mu\frac{du}{dy}$

u is a function of y only and $P + \gamma h$ is a function of l only

$\therefore$ We can change the partial to derivative

5-2 Laminar Incompressible: Steady flow between parallel plate

$$\frac{\partial \tau}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} = \frac{d}{\partial l}(p+\gamma h)$$

or $$\mu \frac{d^2 u}{\partial y^2} = \frac{d}{\partial l}(p+\gamma h)$$

integrating with respect to y we get

$$\mu \frac{du}{dy} = y \frac{d}{\partial l}(p+\gamma h) + A$$

and $$\boxed{u = \frac{1}{2\mu}\frac{d}{\partial l}(p+\gamma h) y^2 + \frac{A}{\mu} y + B} \quad \text{--- 5-1}$$

Boundary conditions

① at $y=0$ $u=0$

at $y=a$ $u=U$

from B.C. ① we get

$B=0$

from Boundary condition② in equ 5-1

we get

$$U = \frac{1}{2\mu}\frac{d}{\partial l}(p+\gamma h) a^2 + \frac{A}{\mu} a$$

$$A = \frac{\mu U}{a} - \frac{d}{\partial l}(p+\gamma h) a$$

Subsituting A & B in equ 5-1 we get

$$\boxed{u = \frac{Uy}{a} - \frac{1}{2\mu}\frac{d}{\partial l}(p+\gamma h)(ay-y^2)} \quad \text{--- 5-2}$$

$h =$ const

$$Q = \int_0^a u\,dy$$

$$\boxed{Q = \frac{Ua}{2} - \frac{1}{12\mu}\frac{d}{\partial l}(p+\gamma h) a^3} \quad \text{--- 5-3}$$

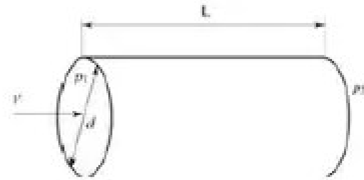
5-3 Laminar and Turbulent flow in pipes

For flow through smooth pipe:

$Re_d < 2300$; flow is laminar
$Re_d > 4000$; flow is turbulent

; Re_d = Reynolds number based on pipe diameter

$$Re_d = \frac{Vd}{\nu}$$

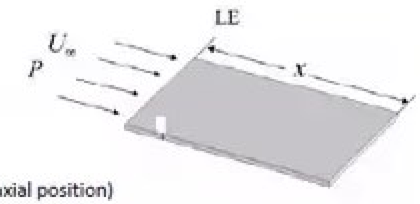


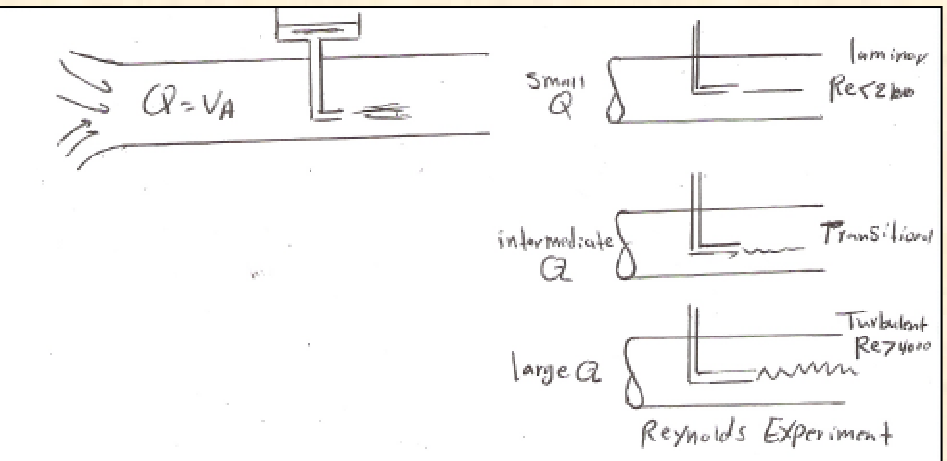
For flow over smooth flat plate:

$Re_x < 1x10^5$; flow is laminar
$Re_x > 5x10^5$; flow is turbulent

; Re_x = Reynolds number based on plate length (/axial position)

$$Re_x = \frac{U_\infty x}{\nu}$$





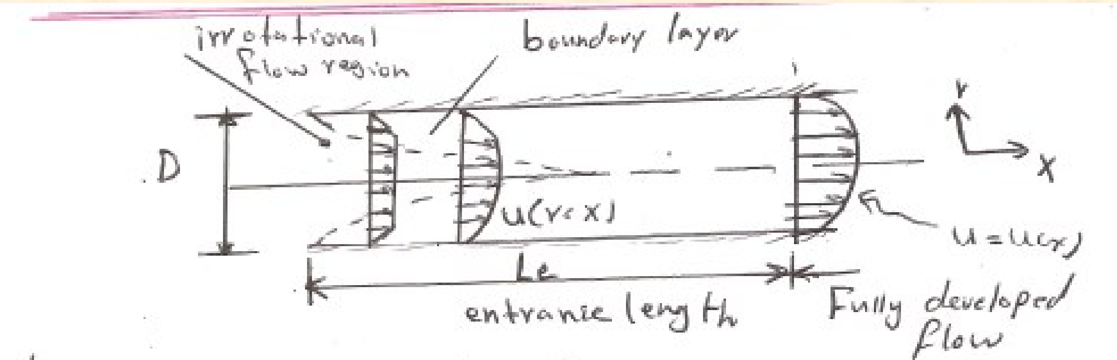
$$N_{re} = \frac{D.V.\rho}{\mu}$$

N_{re}	=	Reynolds Number
μ	=	Viscosity

$N_{re} < 2100$	Flow is laminar
$2100 < N_{re} < 4000$	Flow is in transition
$N_{re} > 4000$	Flow is turbulent

- https://www.quora.com/What-is-the-meaning-of-the-Reynolds-number-Re-in-fluid-dynamics
- https://www.pinterest.com/pin/829014243888125785/

5-4 Entrance Region And Fully Developed



$$\frac{L_e}{D} = 0.06\, Re_D \quad \text{---} \ (5\text{-}5)\ (\text{laminar})$$

$$\frac{L_e}{D} = 4.4\, Re_D^{1/6} \quad \text{---} \ (5\text{-}6)\ (\text{Turbulent})$$

$\tau, \frac{\partial P}{\partial x} = \text{const}$

$\frac{\partial u}{\partial x} = 0$

https://slideplayer.com/slide/17065752/

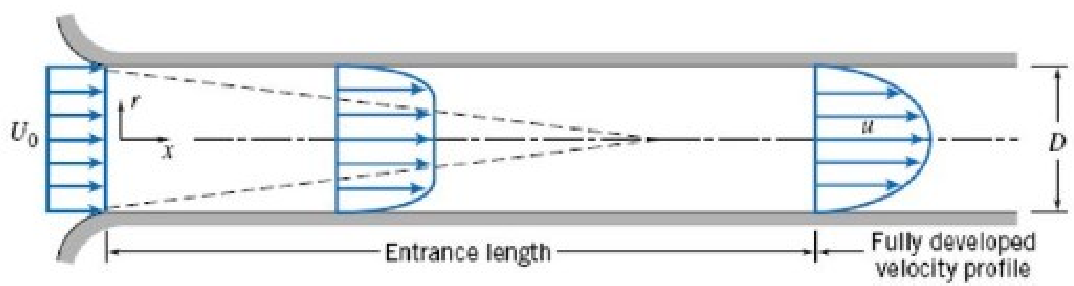


Fig. 8.1 Flow in the entrance region of a pipe.

For laminar flow (Re<2,300)

$$\frac{L}{D} \simeq 0.06 \frac{\rho \bar{V} D}{\mu}$$

Examples

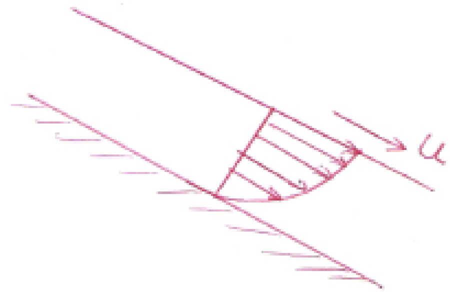
Example$_1$:

Determine the formula for shear stress on each plate of fig when an adverse pressure gradient exists such that $Q = 0$

Solution$_1$:

Sol:-

بما أن مقدار التغير بالضغط
يساوي مقدار التغير بإجهاد
القص



Since

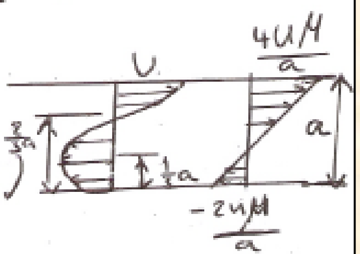
$$Q = \frac{Ua}{2} - \frac{a^3}{12\mu}\frac{d}{dl}(p+\gamma h) = 0$$

$$\therefore \frac{d}{dl}(p+\gamma h) = \frac{6U\mu}{a^2} \quad --- (1)$$

Since

$$u = \frac{Uy}{a} - \frac{1}{2\mu}\frac{d}{dl}(p+\gamma h)(ay-y^2) \quad --- (2)$$

From 1, 2 we get



$$u = \frac{Uy}{a} - \frac{1}{2\mu}\left(\frac{6U\mu}{a^2}\right)(ay-y^2)$$

$$u = 3U\frac{y^2}{a^2} - 2U\frac{y}{a} \quad --- (3)$$

Since $\tau = \mu \frac{du}{dy}$

$$\therefore \tau = \mu\left[3U \cdot \frac{2y}{a^2} - \frac{2U}{a}\right]$$

at $\tau_{y=0} = -\frac{2U\mu}{a}$ at $\tau_{y=a} = \frac{4U\mu}{a}$

Examples

Example$_2$:

Determine the formula for angle θ for fixed parallel plates so that laminar flow at constant pressure takes place?

Solution$_2$:

Sol:-

Constant pressure $\Rightarrow \frac{dp}{dl} = 0$

$\sin\theta = -\frac{dh}{dl}$

Fixed plate $U = 0$

$$\therefore Q = -\frac{a^3}{12\mu}\left(\gamma \frac{dh}{dl}\right) = \frac{a^3}{12\mu}\gamma \sin\theta$$

$$\boxed{\theta = \sin^{-1}\left(\frac{12\mu Q}{\gamma a^3}\right)}$$

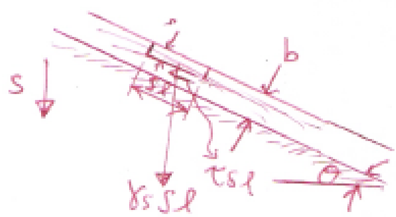
Example$_3$:

With a free body as in Fig for uniform of a thin laminar of liqurd down an inclined plane, show that the velocity distribution is

$$u = \frac{\gamma}{2\mu}(b^2 - s^2)\sin\theta$$

and that the discharge per unit width is

$$Q = \frac{\gamma}{3\mu} b^3 \sin\theta$$



Solution$_3$:

Free surface $dp = 0$

$\Sigma F = 0$

$\gamma s\, dl \sin\theta - \tau\, dl = 0$

$\therefore \tau = \gamma \sin\theta = -\mu \frac{du}{ds} \quad ; s = -y$

$\therefore u = -\frac{\gamma}{\mu}\sin\theta \frac{s^2}{2} + C$

Boundary condition

at $s = b \quad u = 0 \quad \therefore C = \frac{\gamma}{\mu}\sin\theta \frac{b^2}{2}$

$$\therefore u = -\frac{\gamma}{\mu}\sin\theta \frac{s^2}{2} + \frac{\gamma}{\mu}\sin\theta \frac{b^2}{2}$$

$$\boxed{u = \frac{\gamma}{2\mu}\sin\theta(b^2 - s^2)}$$

$$Q = \int_0^b u\, ds = \int_0^b \frac{\gamma}{2\mu}\sin\theta(b^2 - s^2)\, ds$$

$$Q = \frac{\gamma}{2\mu}\sin\theta\left[b^2 s - \frac{s^3}{3}\right]_0^b$$

$$\therefore \boxed{Q = \frac{\gamma}{3\mu} b^3 \sin\theta}$$

- Note:
  - Read example 9-15 in the fluid mechanics book (pp.439-442)
- I hope everything is clear for all students
  - Good luck