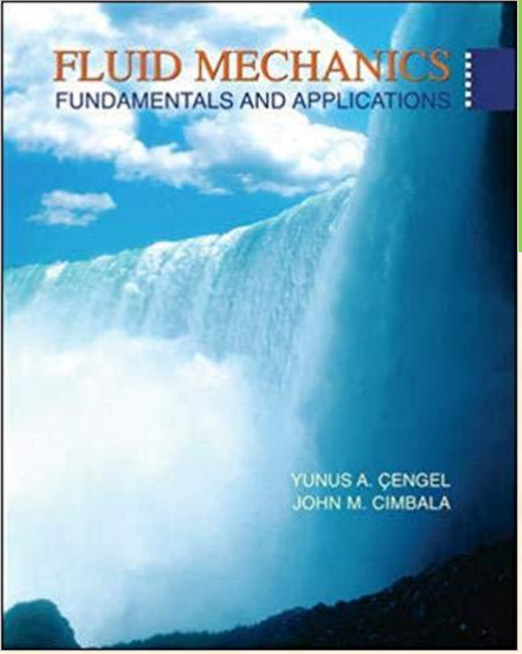

YUNUS A. ÇENGEL and JOHN M. CIMBALA, "Fluid Mechanics: Fundamentals and Applications",1st ed., McGraw-Hill,2006.

Course name

Principles of Fluid Mechanics

Lecture-05- Chapter-02

Fluid Statics: FLUIDS IN RIGID-BODY MOTION

Lecture slides by

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Outline

- ***FLUIDS IN RIGID-BODY MOTION:***
- **Introduction**
- **Case 1: fluid Rest**
- **Case 2: Free Fall of a Fluid Body**
- **Acceleration on a Straight Path**
- ***Relative Equilibrium :***
- **Uniform Linear Acceleration**
- ***Examples 1-3***
- ***Rotation in a Cylindrical Container***
- ***Examples 1-4***
- ***Exams and Grading Policy:***
- ***References***

$$T(\text{K}) = T(^\circ\text{C}) + 273.15$$

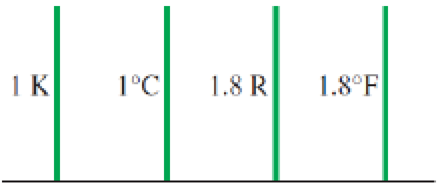
$$T(\text{R}) = T(^\circ\text{F}) + 459.67$$

$$T(\text{R}) = 1.8T(\text{K})$$

$$T(^\circ\text{F}) = 1.8T(^\circ\text{C}) + 32$$

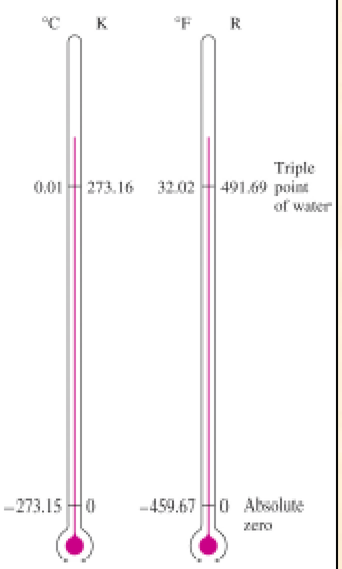
$$\Delta T(\text{K}) = \Delta T(^\circ\text{C})$$

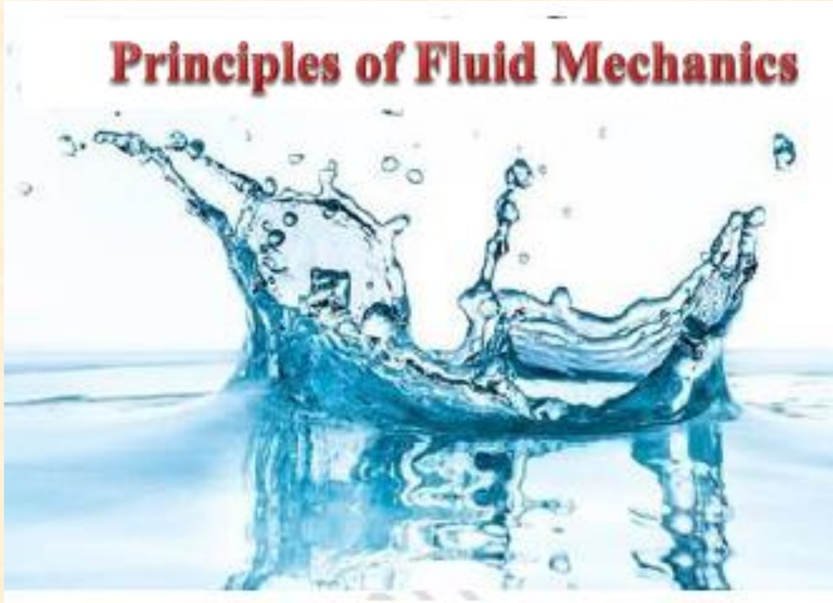
$$\Delta T(\text{R}) = \Delta T(^\circ\text{F})$$



Comparison of magnitudes of various temperature units.

Comparison of temperature scales.

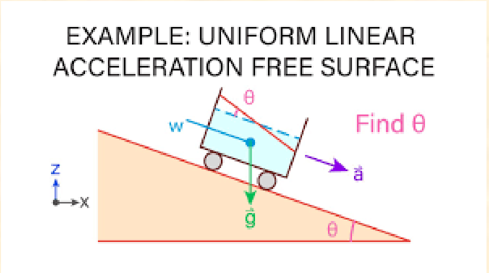


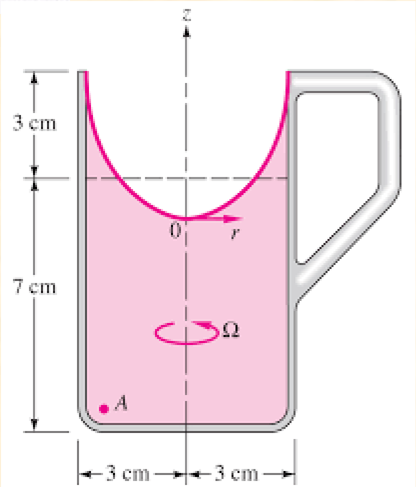


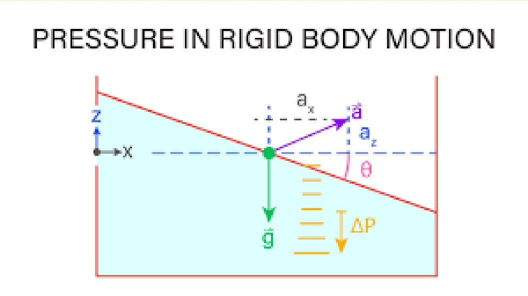
FLUIDS IN RIGID-BODY MOTION: Introduction

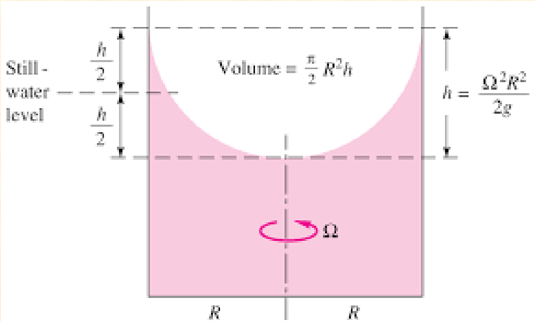
- Many fluids such as milk and gasoline are transported in tankers. In an accelerating tanker, the fluid rushes to the back, and some initial splashing occurs. But then a new free surface (usually no horizontal) is formed, each fluid particle assumes the same acceleration, and the entire fluid moves like a **rigid body**.
- No shear stresses develop within the fluid body since there is no deformation and thus no change in shape.
- Rigid-body motion of a fluid also occurs when the fluid is contained in a tank that rotates about an axis.

Consider a differential rectangular fluid element of side lengths dx, dy, and dz in the x-, y-, and z-directions, respectively, with the z-axis being upward in the vertical direction (Fig. 3–48). Noting that the differential fluid element behaves like a rigid body, Newton's second law of motion for this element can be expressed as:=





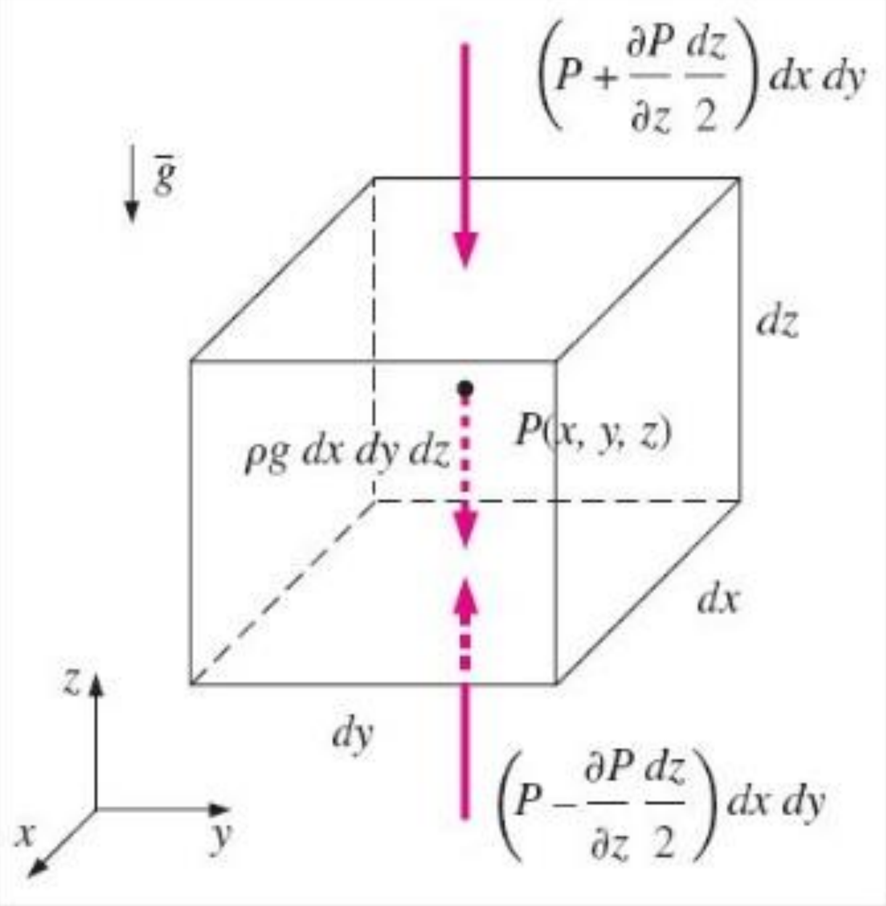




FLUIDS IN RIGID-BODY MOTION: Introduction

Pressure at a given point has the same magnitude in all directions, and thus it is a *scalar* function.

In this section we obtain relations for the variation of pressure in fluids moving like a solid body with or without acceleration in the absence of any shear stresses (i.e., no motion between fluid layers relative to each other).



Newton's 2nd law of motion

$$\delta\vec{F} = \delta m \cdot \vec{a}$$

$$\delta m = \rho\, dV = \rho\, dx\, dy\, dz$$

Net surface force:

$$\delta F_{S,z} = \left(P - \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy - \left(P + \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy = -\frac{\partial P}{\partial z}\, dx\, dy\, dz$$

F=P*A

$$\delta F_{S,x} = -\frac{\partial P}{\partial x}\, dx\, dy\, dz \quad \text{and} \quad \delta F_{S,y} = -\frac{\partial P}{\partial y}\, dx\, dy\, dz$$

FLUIDS IN RIGID-BODY MOTION: Introduction

Expressing in vector form:

$$\delta\vec{F}_S = \delta F_{S,x}\vec{i} + \delta F_{S,y}\vec{j} + \delta F_{S,z}\vec{k}$$

$$= -\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right) dx\,dy\,dz = -\vec{\nabla}P\,dx\,dy\,dz$$

i, j, k = unit vectors in x,y,z directions

$$\vec{\nabla}P = \frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}$$

Pressure gradient

Body force,

$$\delta\vec{F}_{B,z} = -g\delta m\vec{k} = -\rho g\,dx\,dy\,dz\vec{k}$$

← **Remember, m = ρ V**

$$\delta\vec{F} = \delta\vec{F}_S + \delta\vec{F}_B = -(\vec{\nabla}P + \rho g\vec{k})\,dx\,dy\,dz$$

From Newton's 2nd law of motion, δF = δma = ρ dx dy dz . a
Therefore,

Rigid-body motion of fluids: $\nabla P + \rho g k = -\rho\vec{a}$

$$\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k} + \rho g\vec{k} = -\rho(a_x\vec{i} + a_y\vec{j} + a_z\vec{k})$$

Expressing in scalar form in three orthogonal directions;

Accelerating fluids: $\frac{\partial P}{\partial x} = -\rho a_x$, $\frac{\partial P}{\partial y} = -\rho a_y$, and $\frac{\partial P}{\partial z} = -\rho(g + a_z)$

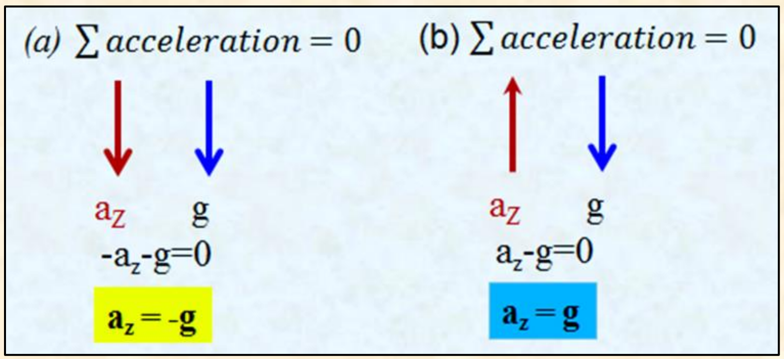
FLUIDS IN RIGID-BODY MOTION: Case 1: fluid Rest

Special Case 1: Fluids at Rest

For fluids at rest or moving on a straight path at constant velocity, all components of acceleration are zero, and the relations reduce to

Fluids at rest: $\frac{\partial P}{\partial x} = 0, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{dP}{dz} = -\rho g$

The pressure remains constant in any horizontal direction (P is independent of x and y) and varies only in the vertical direction as a result of gravity [and thus $P = P(z)$]. These relations are applicable for both compressible and incompressible fluids.



$$\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = -\rho a_y, \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho(g + a_z)$$

- In relative equilibrium of fluid mass, there is no shear stress occure between the bottom layer of fluid & the wall at steady flow (no motion). That mean the τ can be happened at fluid flow (motion).

* For more explaining of fluid motion *

by using (Newton's Second law of motion)

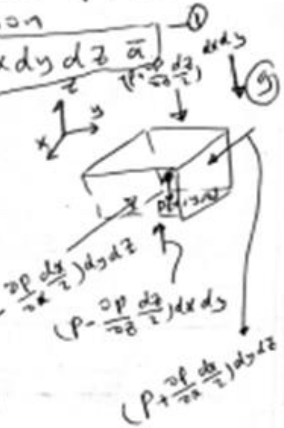
$\Sigma F = m \cdot a$ (Newton's 2nd low of fluid motion) —①

$\delta F = \delta m \cdot \bar{a} \Rightarrow \delta F = \rho \, d\forall \, \vec{a} = \rho \, dx \, dy \, dz \, \bar{a}$

* Horizontal acceleration (in x-direction)

$\delta F_{s,x} = (P - \frac{\partial P}{\partial x}\frac{dx}{2}) dy\,dz - (P + \frac{\partial P}{\partial x}\frac{dx}{2}) dy\,dz$

$\therefore \delta F_{s,x} = -\frac{\partial P}{\partial x} dx\,dy\,dz$ —②



- vertical acceleration: (In z-direction)

$\delta F_{s,z} = (P - \frac{\partial P}{\partial z}\frac{dz}{2}) dx\,dy - (P + \frac{\partial P}{\partial z}\frac{dz}{2}) dx\,dy$

$\therefore \delta F_{s,z} = -\frac{\partial P}{\partial z} dx\,dy\,dz$

$\frac{\partial P}{\partial x} = -\rho a_x, \; \frac{\partial P}{\partial y} = -\rho a_y$

- (acceleration fluid) $\Rightarrow$ Equ.① & ② $\Rightarrow$

& $\frac{\partial P}{\partial z} = -\rho(g + a_z)$

$w = m \cdot g = \rho \forall g$

$w = \rho g \, dx\,dy\,dz$

- (Fluid at rest): $\frac{\partial P}{\partial x} = 0, \frac{\partial P}{\partial y} = 0$ & $\frac{\partial P}{\partial z} = -\rho g$

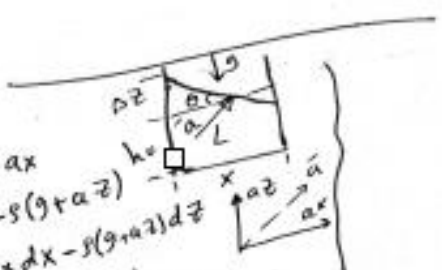
ex

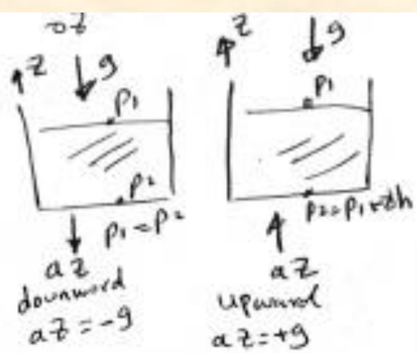
$\frac{\partial P}{\partial x} = -\rho a_x$

$\frac{\partial P}{\partial z} = -\rho(g + a_z)$

$dP = -\rho a_x dx - \rho(g + a_z) dz$

$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1)$





Special Case 2: Free Fall of a Fluid Body

A freely falling body accelerates under the influence of gravity. When the air resistance is negligible, the acceleration of the body equals the gravitational acceleration, and acceleration in any horizontal direction is zero. Therefore, $a_x = a_y = 0$ and $a_z = -g$.

Free-falling fluids: $$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial z} = 0 \quad \rightarrow \quad P = \text{constant}$$

$$\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = -\rho a_y, \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho(g + a_z)$$

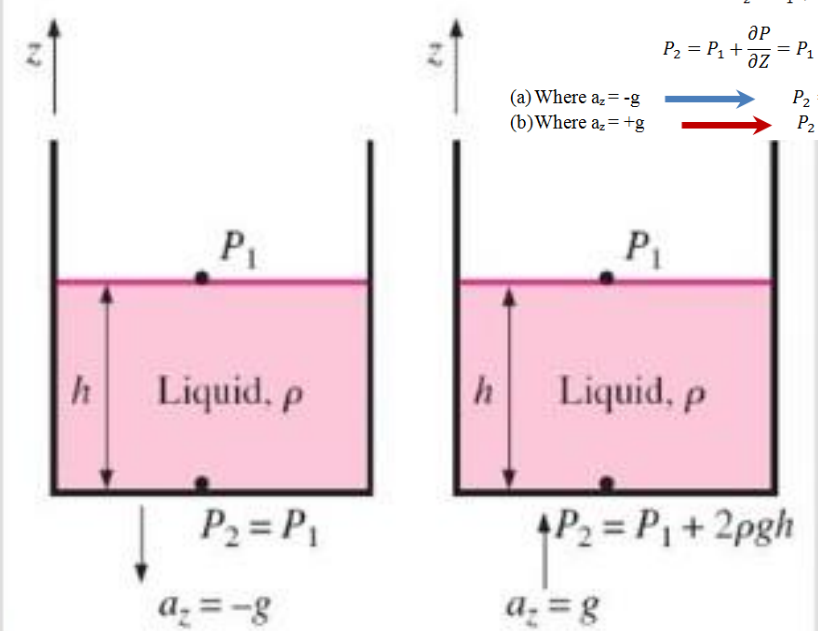
$$P_2 = P_1 + P_{gage}$$

$$P_2 = P_1 + \frac{\partial P}{\partial z} = P_1 + \rho(g + a_z)h$$

(a) Where $a_z = -g$ → $P_2 = P_1$

(b) Where $a_z = +g$ → $P_2 = P_1 + 2\rho gh$

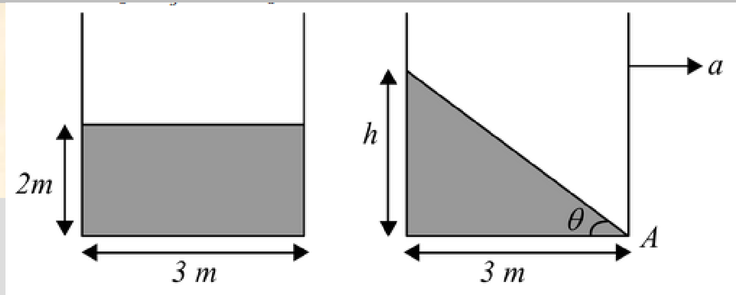
(a) $\sum acceleration = 0$ (b) $\sum acceleration = 0$



(*a*) Free fall of a liquid

(*b*) Upward acceleration of a liquid with $a_z = +g$

The effect of acceleration on the pressure of a liquid during free fall and upward acceleration.



Acceleration on a Straight Path

Example: a container partially filled with a liquid, moving on straight path with constant acceleration.

a_y = 0, then Equation 3-43 reduced to:

$$\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho(g + a_z)$$

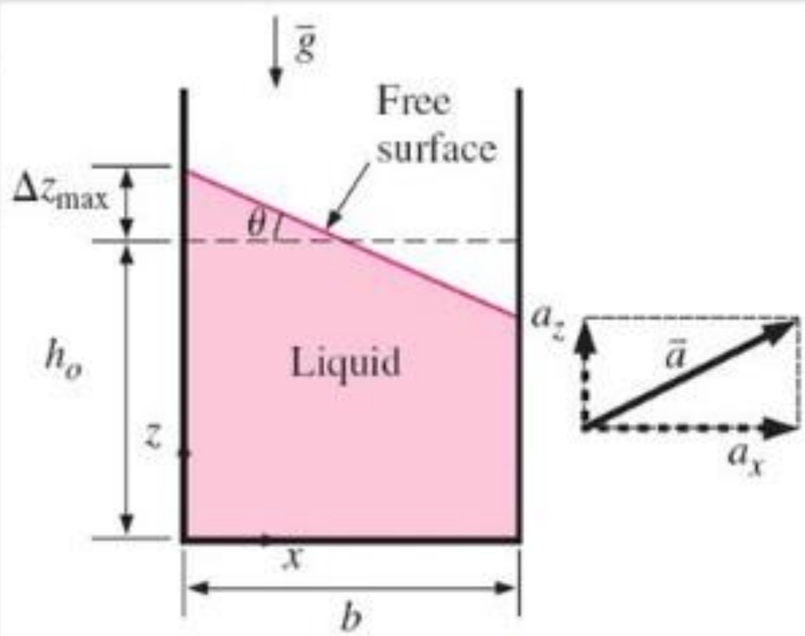
$$P = P(x, z)$$

Therefore:

$$dP = (\partial P/\partial x)\, dx + (\partial P/\partial z)\, dz$$

$$dP = -\rho a_x\, dx - \rho(g + a_z)\, dz$$

$$P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1)$$

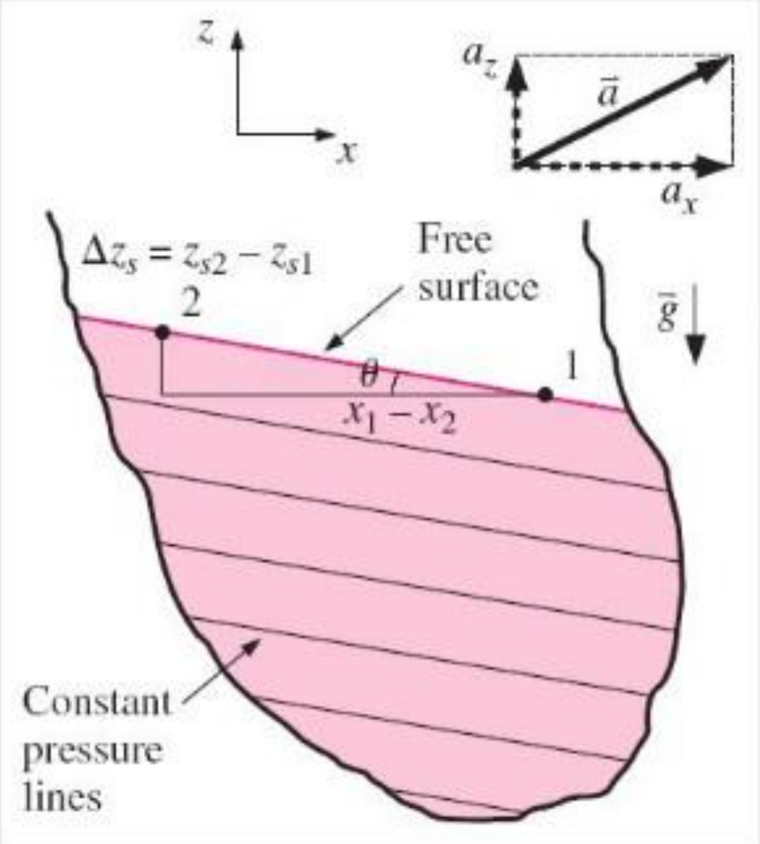


Rigid-body motion of a liquid in a linearly accelerating tank.

Taking point 1 to be origin (x1 = 0, z1 = 0),

Pressure variation: $P = P_0 - \rho a_x x - \rho(g + a_z)z$

FLUIDS IN RIGID-BODY MOTION: Acceleration on a Straight Path



Lines of constant pressure (which are the projections of the surfaces of constant pressure on the *xz*-plane) in a linearly accelerating liquid. Also shown is the vertical rise.

$$P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1)$$

By taking P1 = P2,

Vertical rise of surface:

$$\Delta z_s = z_{s2} - z_{s1} = -\frac{a_x}{g + a_z}(x_2 - x_1)$$

z_s = z-coordinate of liquid's free surface

Setting dP = 0,

Surfaces of constant pressure: *(isobars)*

$$\frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = \text{constant}$$

Slope of isobars:

$$\text{Slope} = \frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta$$

Relative Equilibrium : Uniform Linear Acceleration

For steady flow, mass in motion no shear stress will occur if there is no relative motion between adjacent layer of the fluid.

(a) Horizontal Acceleration

$$\sum F_x = m.a$$

$$P_1 d_A - P_2 d_A = \frac{\gamma}{g} l d_A a_x \quad \div \gamma\, d_A$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} = \frac{l\, a_x}{g}$$

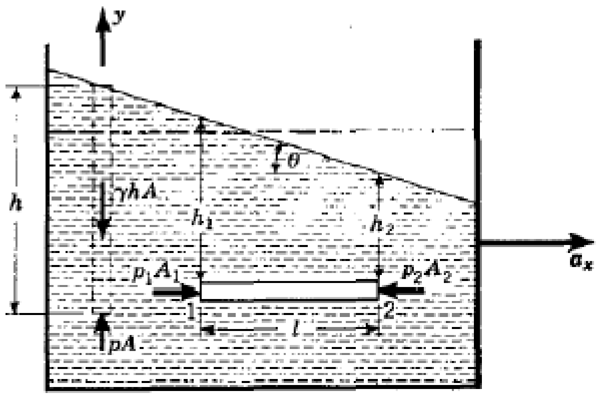
$$h_1 - h_2 = \frac{l\, ax}{g} \qquad \div l$$

Or

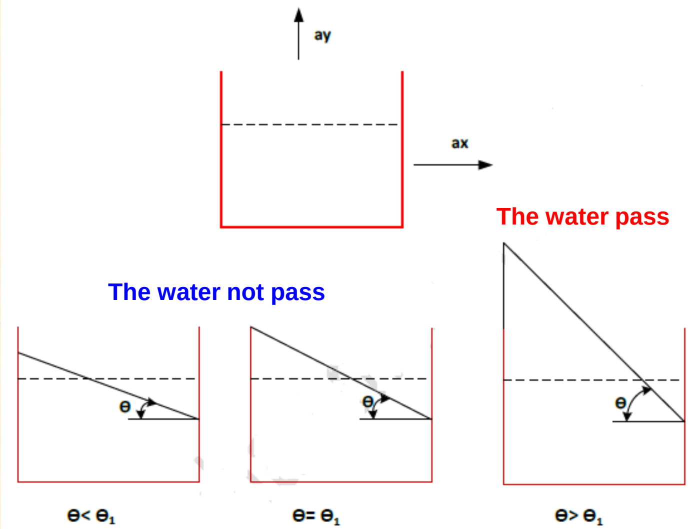
$$\frac{h_1 - h_2}{l} = \frac{a_x}{g}$$

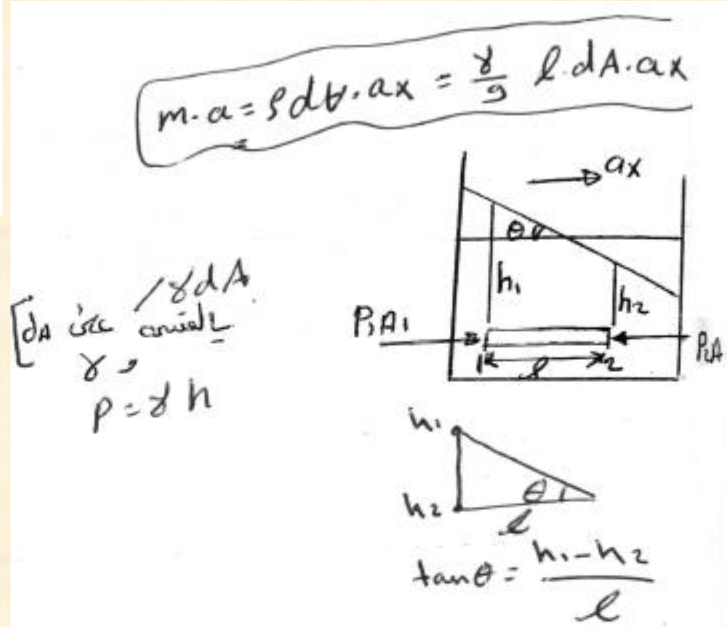
From fig. the left side is the slop

$$\tan\theta = \frac{a_x}{g}$$



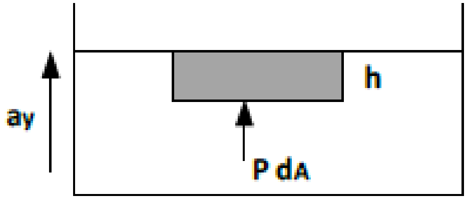
Note





(b) Vertical Acceleration

$$\sum F_y = ma_y$$



$$Pd_A - \gamma h d_A = \frac{\gamma}{g} h\, d_A a_y$$

$$P = \gamma h\left(1 + \frac{a_y}{g}\right) \ Upward$$

$$P = \gamma h\left(1 - \frac{a_y}{g}\right) \ Downward$$

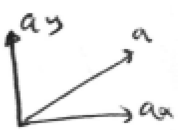
The general equation for a tank moved in two direction X &Y

a_x: The acceleration in X-direction

a_y: The acceleration in Y-direction

P_o: The initial pressure and equal to atmospheric pressure when the tank is opened

$$P = P_o - \gamma\frac{a_x}{g}X - \gamma\left(1 + \frac{a_y}{g}\right)Y$$



and

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

$$\tan\theta = -\frac{a_x}{a_y+g}$$

$$\tan\theta = -\frac{a_x}{a_y+g}$$

θ:ميل السطح

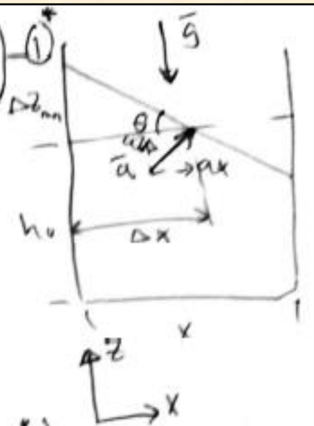
الاشارة السالبة تعني ان الزاوية في الربع الثاني

$$\therefore P_2 - P_1 = -\rho a_x(X_2 - X_1) - \rho(g + a_z)(Z_2 - Z_1) \quad (1)$$

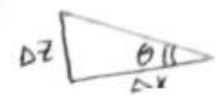
$$\therefore P = P_o - \rho a_x X - \rho(g + a_z) Z$$

$$\gamma = \rho g \Rightarrow \rho = \frac{\gamma}{g}$$

$$P = P_o - \gamma\frac{a_x}{g}X - \gamma\left(1 + \frac{a_z}{g}\right)Z$$



$$slop = \tan\theta = \frac{\Delta Z}{\Delta X}$$ (by using eq (1) where $\Delta P = 0$)



$$\Rightarrow \tan\theta = \frac{\Delta Z}{\Delta X} = -\frac{\rho a_x}{\rho(g + a_z)}$$

$$\therefore \tan\theta = -\frac{a_x}{a_z + g}$$

Examples:

Example $_1$: The tank is accelerated in the X-direction in such a way that the liquid surface doesn't change slope What is the acceleration of the tank?

Sol.$_1$:

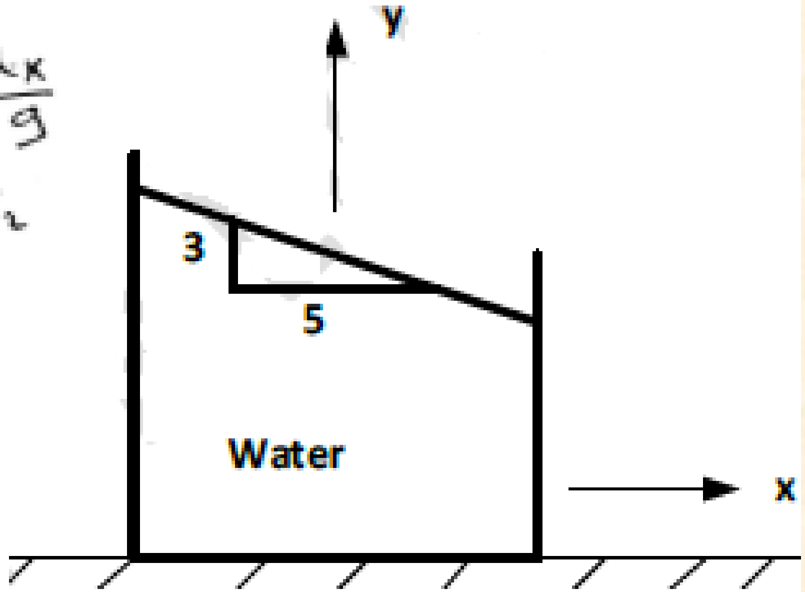
$$\tan\theta = \frac{a_x}{g}$$

$$\frac{3}{5} = \frac{a_x}{9.81}$$

$a_x = 5.886\ m/s^2$

in x- direction $\tan\theta = \frac{a_x}{g}$

$\therefore \frac{3}{5} = \frac{a_x}{g} \Rightarrow a_x\ \frac{m}{s^2}$



Examples:

Example 2: In the below fig. a_x=3.9 m/s^2 ,a_y=0.Find the imaginary free liquid surface and the pressure at B,C,D and E.?

Sol.2:

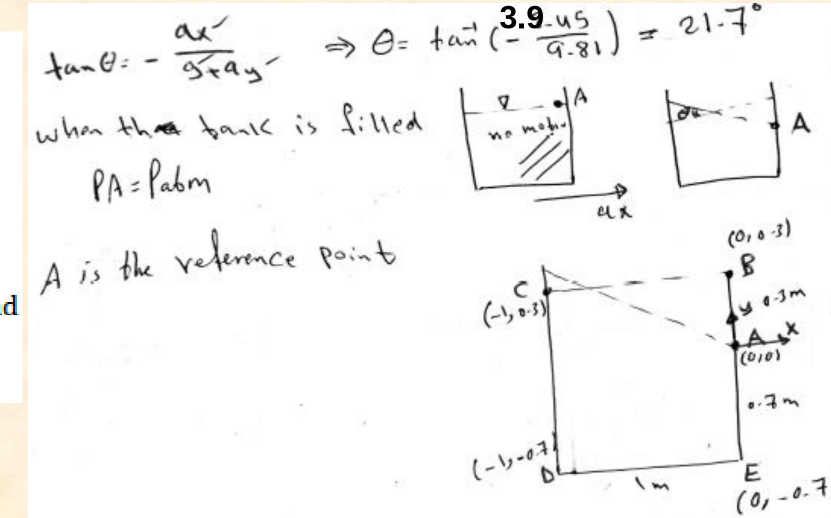
Imaginary free

$$\tan\theta = -\frac{a_x}{a_y + g}$$

$$= -\frac{3.9}{9.81} = 21.7^o$$

Since the tank is filled and

$$P_A = P_{atm}$$



$$P = P_o - \gamma\frac{a_x}{g}X - \gamma\left(1 + \frac{a_y}{g}\right)Y$$

$$P = -0.8 * 9810 * \frac{3.9}{9.81}X - 0.8 * 9810 * \left(1 + \frac{0}{g}\right)Y$$

$$P = -3120\,X - 7848\,Y$$

At point B

X=0, Y=0.3

$$P_B = 0 - 7848 * 0.3$$

P_B=-2.354 Kn/m^2

At point C

X=-1, Y=0.3

P_C=0.7656 Kpa

At point D

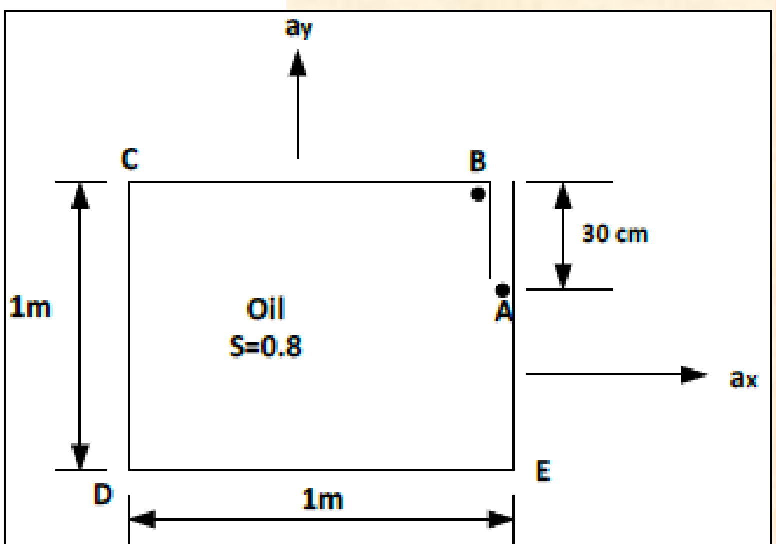
X=-1, Y=-0.7

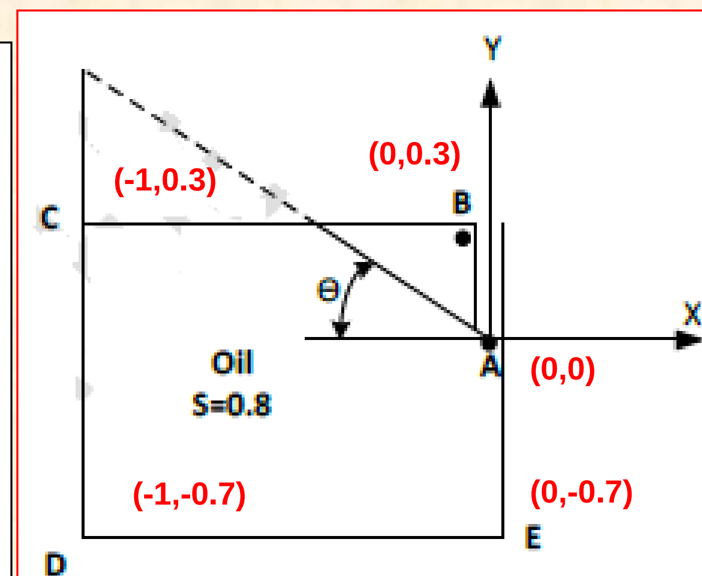
P_D=8.614 Kpa

At point E

X=0,Y=-0.7

P_E=5.494 Kpa





Examples:

Example $_3$***:*** In the fig a_x=9.806 m/s^2 ,,a_y=0.Find the imaginary free liquid surface and the pressure at A,B,C.?

Sol.$_3$:

Volume of the initial space = volume of new space

$$0.3 * 1.3 = x * \frac{h}{2} = x^2 * \frac{\tan\theta}{2}$$

X=0.883176 m

h=0.883 m

$$\tan\theta = \frac{h+5}{1.3} = 1 \rightarrow s = 1.3 - 0.883 = 0.4168\ m$$

$$P = P_o - \gamma\frac{a_x}{g}X - \gamma\left(1+\frac{a_y}{g}\right)Y$$

P= -9810X-9810 Y

At point A

X=-1.3 , Y=0.8831

P_A=4.09 Kpa

At point B

X=-1.3 , Y=-0.4168

P_B=16.842 Kpa

At point C

X=0, Y=-0.4168

P_C=4.09 Kpa

$$\tan\theta = -\frac{a_x}{a_y+g}$$

$$\tan\theta = -\frac{9.806}{0+9.806} = -1 \rightarrow \theta = 45^o$$

$$\tan\theta_1 = \frac{0.3}{0.65} \rightarrow \theta_1 = 24.74^o$$

$\theta > \theta_1$

The water passes point A

$\tan\theta = \frac{h}{x} \rightarrow h = x\tan\theta$..........1

$\tan\theta = \frac{a_x}{a_y+g} \Rightarrow \theta = 45°$

$\tan\theta_1 = \frac{h}{x} \Rightarrow h = x\tan\theta_1$ —(1)

$V_1 = V_2$

$0.3 \times 1.3 = \frac{1}{2}x \cdot h$ —(2)

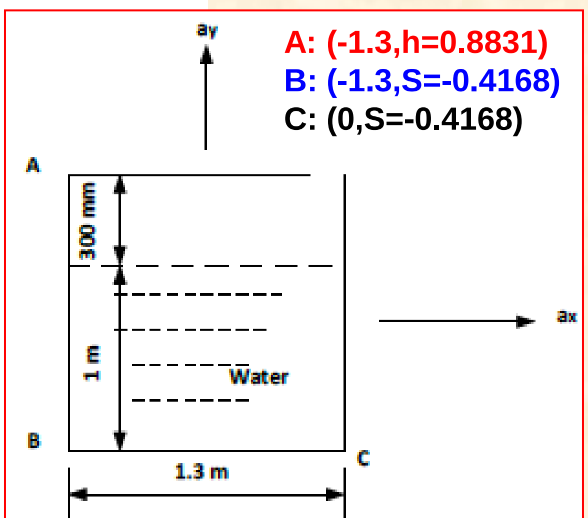
Substitute h of eq(1) in eq(2)

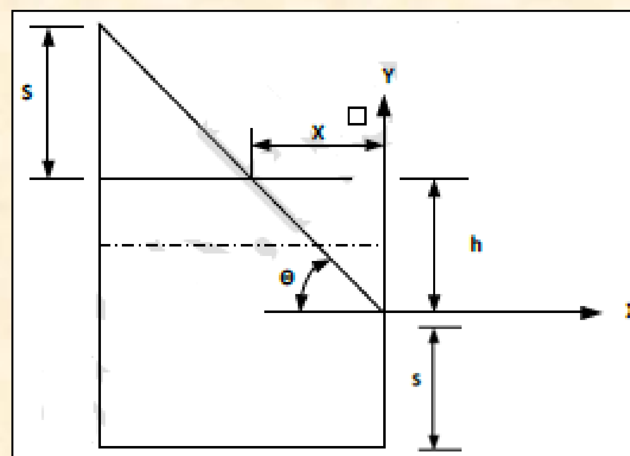
$\therefore 0.3 \times 1.3 = \frac{1}{2}x(x\tan\theta)$ (45°) $\Rightarrow x = 0.883$ m, h=0.883 m

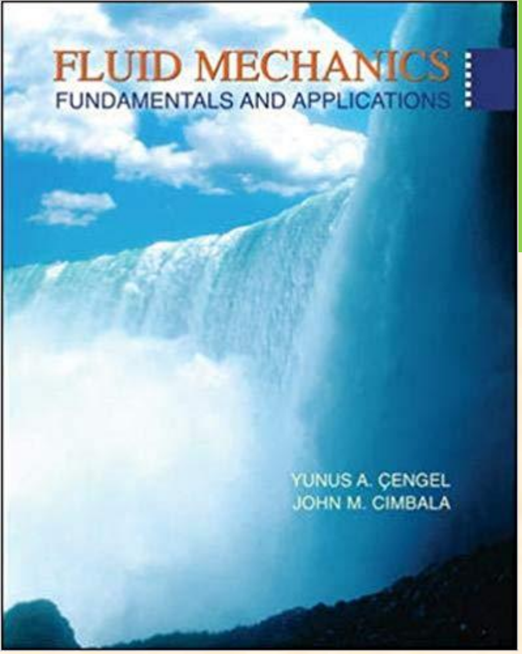
To find the distance s

$\tan\theta = \frac{S+h}{1.3} \Rightarrow S = 0.41168$

$P = P_o - \gamma\frac{a_x}{g}x - \gamma(1+\frac{a_y}{g})y$







YUNUS A. ÇENGEL and JOHN M. CIMBALA, "Fluid Mechanics: Fundamentals and Applications",1st ed., McGraw-Hill,2006.

Course name

Principles of Fluid Mechanics

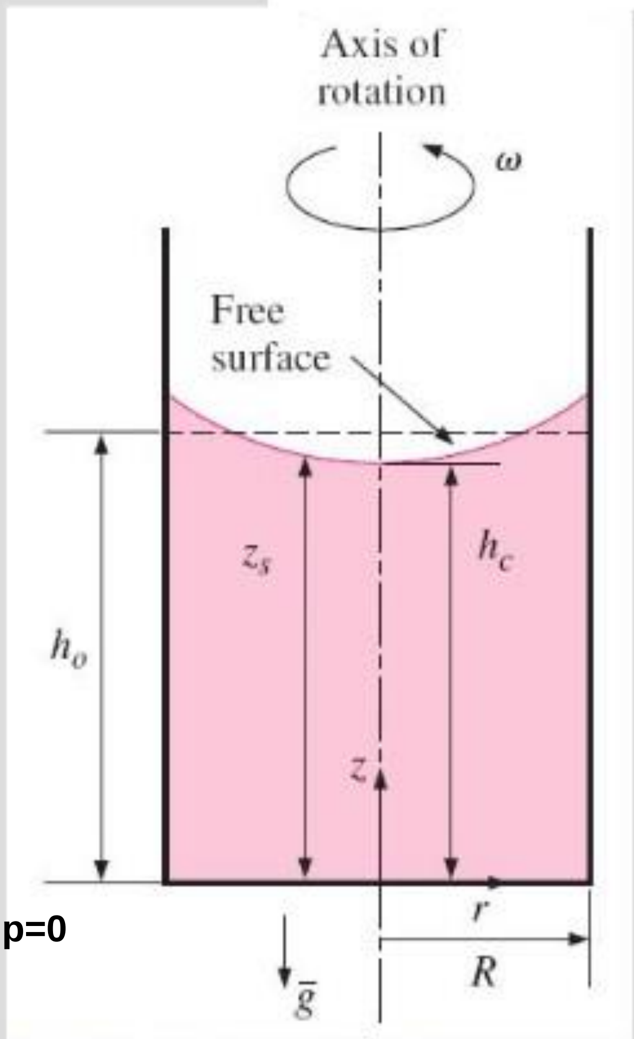
Lecture-05-1- Chapter-02

Fluid Motion: Rotation in a Cylindrical Container

Lecture slides by
Assistant Professor Dr. Thamer Khalif Salem
University of Tikrit

Rotation in a Cylindrical Container

Consider a vertical cylindrical container partially filled with a liquid. The container is now rotated about its axis at a constant angular velocity of ω. After initial transients, the liquid will move as a rigid body together with the container. There is no deformation, and thus there can be no shear stress, and every fluid particle in the container moves with the same angular velocity.



Rigid-body motion of a liquid in a rotating vertical cylindrical container.

Equations of motion for rotating fluid;

$$\frac{\partial P}{\partial r} = \rho r \omega^2, \qquad \frac{\partial P}{\partial \theta} = 0, \qquad \text{and} \qquad \frac{\partial P}{\partial z} = -\rho g$$

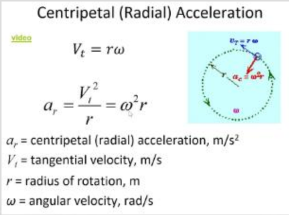
$$P = P(r, z) \qquad dP = (\partial P/\partial r)dr + (\partial P/\partial z)dz$$

$$dP = \rho r \omega^2\, dr - \rho g\, dz$$

At P=const. , dp=0

$$\frac{dz_{\text{isobar}}}{dr} = \frac{r\omega^2}{g}$$

Integrating:



Surfaces of constant pressure: $z_{\text{isobar}} = \frac{\omega^2}{2g} r^2 + C_1$

$a_r = V^2/r = (r^* \omega)^2/r = r^* \omega^2$

FLUIDS IN RIGID-BODY MOTION: Uniform Rotational Vortex flow

Paraboloid: الجسم المكافئ الدوراني

$$z_s = \frac{\omega^2}{2g} r^2 + h_c$$

The distance of free surface from the bottom of container at radius r

$$V = \int_{r=0}^{R} 2\pi z_s r\, dr = 2\pi \int_{r=0}^{R} \left(\frac{\omega^2}{2g} r^2 + h_c\right) r\, dr = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_c\right)$$

Volume of paraboloid formed by the free surface

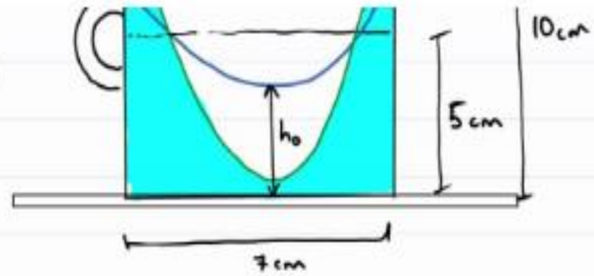
$$V = \pi R^2 h_0$$

Original volume of fluid in container (without rotation) **At ω =0**

$$h_c = h_0 - \frac{\omega^2 R^2}{4g}$$

Free surface: $z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$

$$h = \frac{\omega^2 r^2}{2g} + h_0$$



$$V = \pi R^2 H \qquad V = \pi r^2 h$$

$$\partial V = 2\pi r h\, dr$$

$$\therefore V = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_c\right)$$

$$\pi R^2 h_0 = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_c\right)$$

$$\therefore h_c = h_0 - \frac{\omega^2 R^2}{4g}$$

$$\therefore z_s = \frac{\omega^2 r^2}{2g} + h_c$$

$$= \frac{\omega^2 r^2}{2g} + h_0 - \frac{\omega^2 R^2}{4g}$$

$$\therefore z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2)$$

Δz_{max} at $r = R$ & $r = 0$

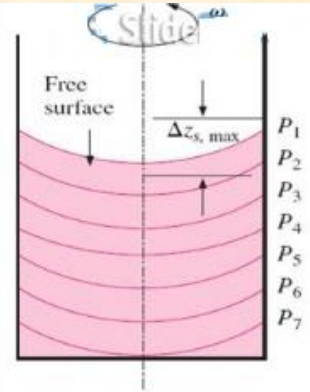
$$z_s(r=R) = h_0 - \frac{\omega^2}{4g}(R^2 - 2R^2) = h_0 + \frac{\omega^2 R^2}{4g}$$

$$z_s(r=0) = h_0 - \frac{\omega^2}{4g}(R^2 - 0) = h_0 - \frac{\omega^2 R^2}{4g}$$

$$\therefore \Delta z_{max} = z_s(R) - z_s(0) = h_0 + \frac{\omega^2 R^2}{4g} - h_0 + \frac{\omega^2 R^2}{4g}$$

$$\therefore \Delta z_{max} = \frac{\omega^2 R^2}{2g}$$

Maximum height difference: $\Delta z_{s,\max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g} R^2$



$$dP = \rho r \omega^2\, dr - \rho g\, dz$$

↓ **integrating:**

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

↓ **Taking point 1 to be the origin (r1=0, z1=0);**

Pressure variation: $P = P_0 + \frac{\rho\omega^2}{2} r^2 - \rho g z$

Note that at a fixed radius, the pressure varies hydrostatically in the vertical direction, as in a fluid at rest.

For a fixed vertical distance *z*, the pressure varies with the square of the radial distance *r*, increasing from the centerline toward the outer edge.

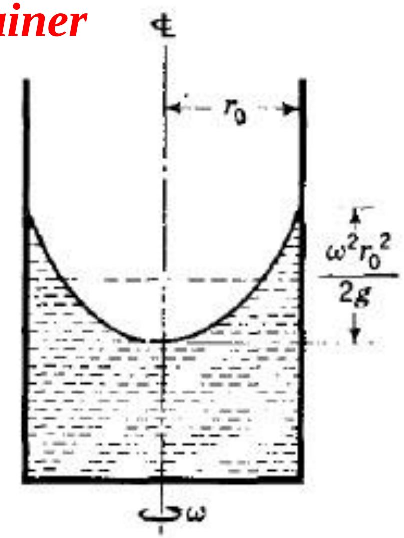
In any horizontal plane, the pressure difference between the center and edge of the container of radius *R* is

$$\Delta P = \rho\omega^2 R^2/2$$

FLUIDS IN RIGID-BODY MOTION: Uniform Rotational Vortex flow

Consider liquid rotating about the central axises with angular velocity (ω) rad/Sec. The slope of water caused by normal acceleration (a_n) and the gravitational acceleration (g).

Rotation in a Cylindrical Container



$$Slope = \frac{dh}{dr} = \frac{a_n}{g}$$

$$dh = \frac{a_n}{g} d_r$$

Since $a_n = \omega^2 r$

$$dh = \frac{\omega^2 r}{g} d_r$$

$$h = \frac{\omega^2 r^2}{2g} + C$$

at $r=0 \longrightarrow h=0$

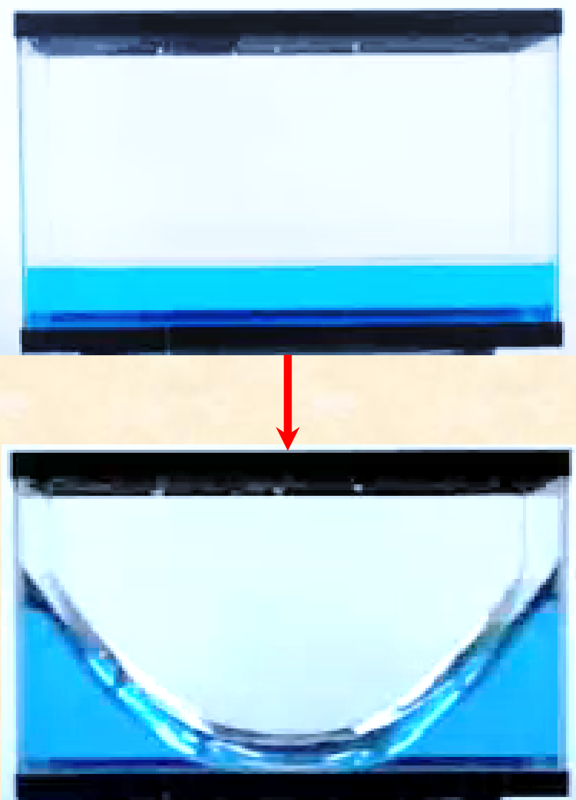
at $r=0$ $h=0$
$\therefore C=0$

$C=0$

$$h = \frac{\omega^2 r^2}{2g}$$

When $P = P_o + \gamma \frac{a_r}{g} r - \gamma (1 + \frac{a_y}{g}) y$ ($\frac{\omega^2 r^2}{2g}$)

$$P = P_o + \frac{\gamma \omega^2 r^2}{2g} - \gamma Y$$



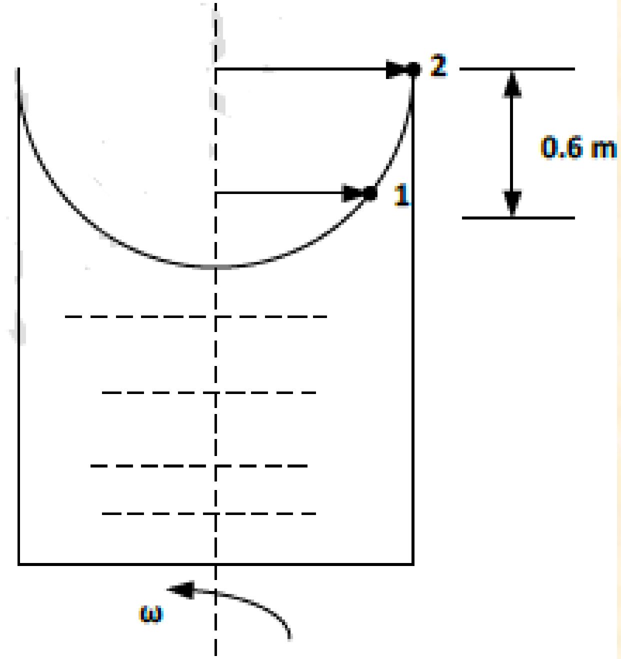
$\frac{\gamma \omega^2 r^2}{2g}$ تمثل التغير بالضغط باتجاه r

γY تمثل التغير بالضغط باتجاه *y-dir*

Examples:

Example $_1$: A vessel containing liquid δ =1.2 is rotated about a vertical axis. The pressure at one point 0.6 m radially from the axis is the same as at another point 1.2 m from the axis and with elevation 0.6 .Calculate the rotational speed.?

$r_1 = 0.6\,m$, $r_2 = 1.2\,m$



$$h = \frac{\omega^2 r^2}{2g}$$

$$h_1 = \frac{\omega^2 r_1^2}{2g}$$

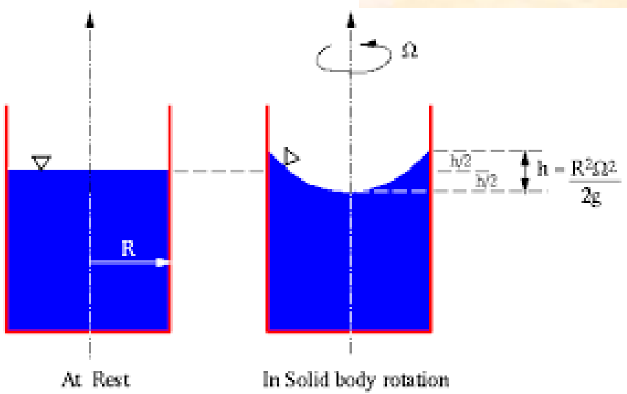
$$h_2 = \frac{\omega^2 r_2^2}{2g}$$

$$h_2 - h_1 = 0.6$$

$$\frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g} = 0.6$$

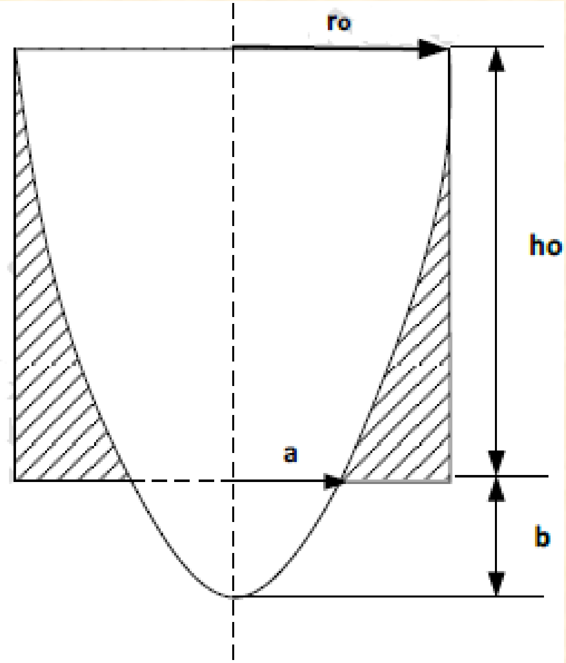
$$0.6 = \frac{\omega^2}{2g}(1.2^2 - 0.6^2) \rightarrow \omega = 3.3\ r/s$$

$$\omega = \frac{2\pi N}{60} \rightarrow N = 31.5\ rpm$$



Examples:

Example $_{2}$***:*** right –circular cylinder of radius r_o and height h_o with axis vertical is open at the top and filled with liquid. At what speed must it rotate so **that half the area of the bottom is exposed**.? **Sol.$_2$:**



$$h = \frac{\omega^2 r^2}{2g}$$

$$b = \frac{\omega^2 a^2}{2g}$$

$$b + ho = \frac{\omega^2 r_o^2}{2g}$$

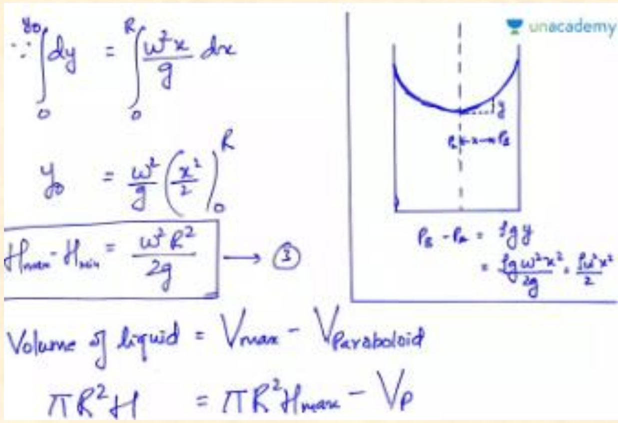
$$ho = \frac{\omega^2}{2g}(r_o^2 - a^2)$$

$$Since \quad \pi a^2 = \frac{1}{2}\pi r_o^2$$

$$a = \frac{r_o}{\sqrt{2}}$$

$$ho = \frac{\omega^2}{2g}\left(r_o^2 - \frac{r_o^2}{2}\right)$$

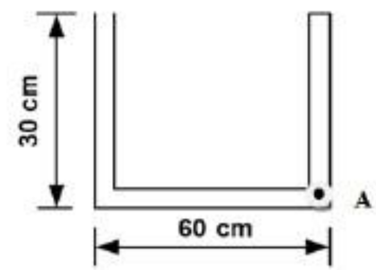
$$\omega = \frac{2\sqrt{gho}}{r_o}$$

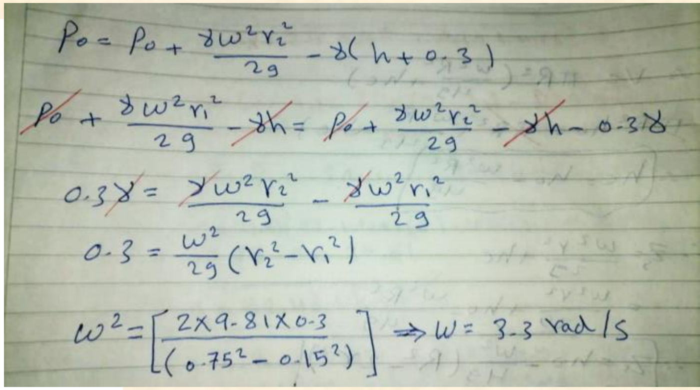


Examples:

Example 3: The U tube of fig is rotated about a vertical axis 15 cm to the right of A at such a speed that the pressure at A is zero gauge. What is the rotational speed.?

Sol.3:





$r_1 = 0.15\ m,$

$r_2 = 0.75m$

$$P = P_o + \frac{\gamma\omega^2 r^2}{2g} - \gamma Y$$

At Point A

$P_A = P_o$

$$P_o = P_o + \frac{\gamma\omega^2 r_1^2}{2g} - \gamma h$$

At Point O

$$P_o = P_o + \frac{\gamma\omega^2 r_2^2}{2g} - \gamma(h+0.3)$$

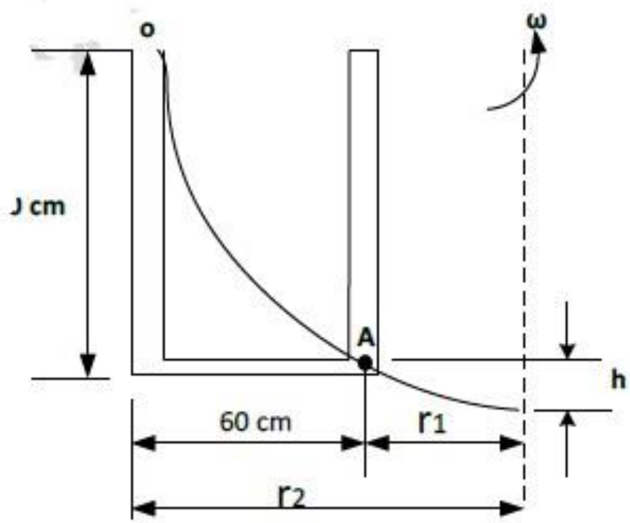
$$\gamma\omega^2 r_1^2 - \gamma h = \gamma\omega^2 r_2^2 - \gamma h - 0.3\gamma$$

$$0.3 = \frac{\omega^2}{2g}(r_2^2 - r_1^2)$$

$$\frac{\omega^2}{2 * 9.81} = (0.75^2 - 0.15^2)$$

$\omega = 3.3\ r/s$

$$\omega = \frac{2\pi N}{60} \rightarrow N = 31.5\ rpm$$



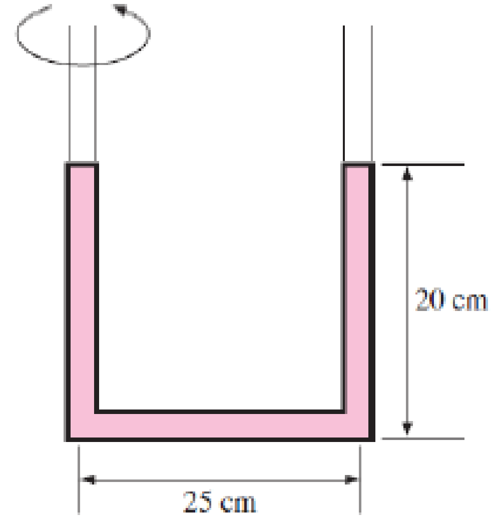
Example $_4$: The distance between the centers of the two arms of a U-tube open to the atmosphere is 25 cm, and the U-tube contains 20-cm-high alcohol in both arms. Now the U-tube is rotated about the left arm at 4.2 rad/s. Determine the elevation difference between the fluid surfaces in the two arms.?

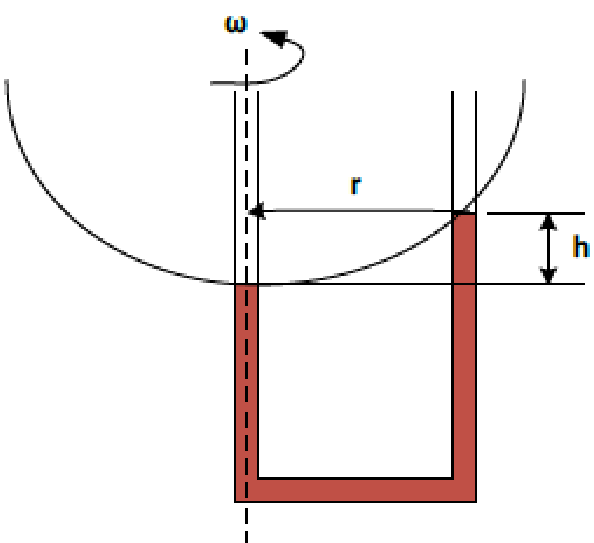
Sol.$_4$:

$$h = \frac{\omega^2 r^2}{2g}$$

$$h = \frac{4.2^2 0.25^2}{2 * 9.81}$$

$$h=0.056\ m$$





Exams and Grading Policy:

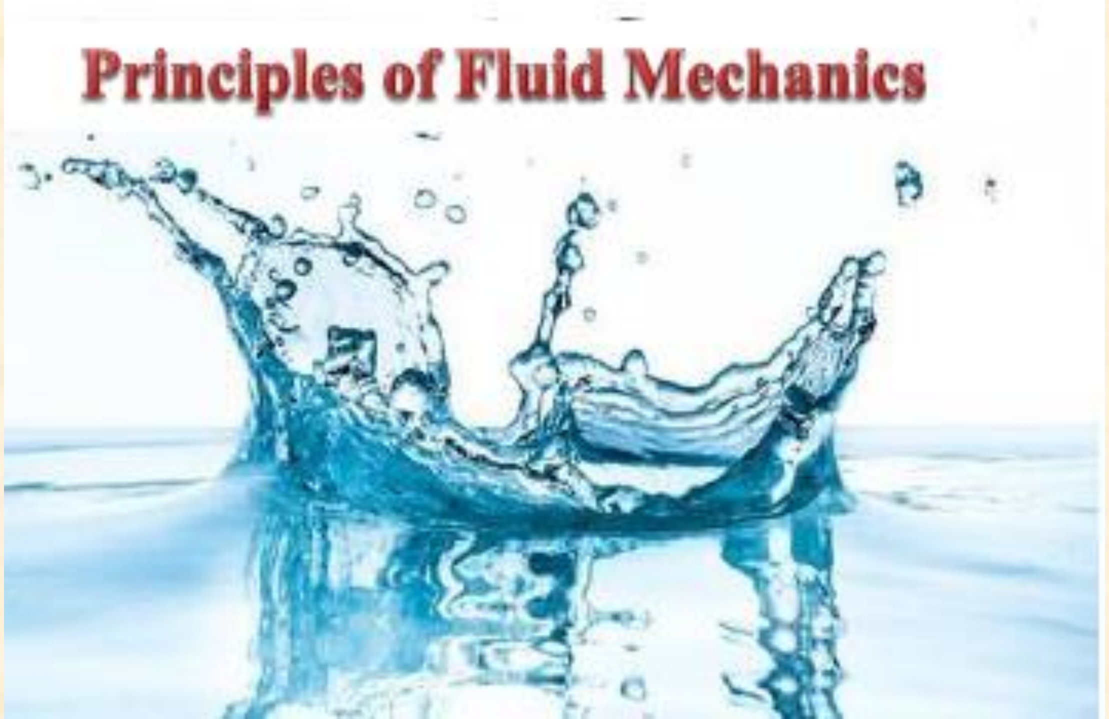
- The distribution of Fluid mechanics degree for the students in Course-1 as following the table:

Exam 20%		Report 10%		Extra Degree 5%			Final Exam 50%	Laboratory 15%			Final
(1)	(2)	Report Structures	Report Discussion	Contribution	Homework	Quizzes	-	**Report Structures**	**Report Discussion**	**Experimental Contribution or final exam**	
10 %	**10 %**	7 %	3 %	**2 %**	2 %	3 %	35 %	7 %	3 %	5 %	**50 %**

References:

References:

- Fluid Mechanics Fundamentals and applications By Yunus A.Cengel and John M.Cimbala.
- Fluid Mechanics,By Streeter,Victor L.
- Fluid Mechanics with Engineering Applications By Robert L.Daugherty.



- Note: ~~Solve all five Homeworks and sending me the answering next week on Sunday 26 October 2022.~~
- In the Fluid book, you have to read the examples (**3.12** and **3.13**) very carefully
- I hope everything is clear for all students

❖Good luck