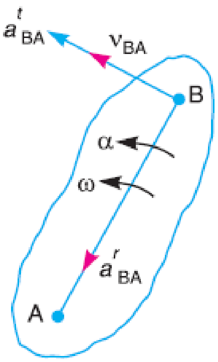
3. Acceleration in Mechanism

We have discussed the velocity diagram of various points in the previous chapter. The acceleration will be discussed in this chapter.

3.1. Acceleration Diagram for a link:

Consider two points A and B on a rigid link as shown in figure below in which point B moves with respect to A, with an angular velocity $\omega\ rad/s$, and let $\alpha\ rad/s^2$ be the angular acceleration of the link AB.



We have already discussed that acceleration of a particle whose velocity changes both in magnitude and direction at any instant has the following two components:

1. The ***centripetal or radial component,*** which is perpendicular to the velocity of the particle at the given instant.

2. The ***tangential component,*** which is parallel to the velocity of the particle at the given instant.

$$a^r_{AB} = \omega^2 \cdot Legth\ of\ link\ AB(is\ always \mathbin{/\!/} AB) = \omega^2 \cdot AB = \frac{v^2_{AB}}{AB}, (\omega = \frac{v_{AB}}{AB})$$

This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts ***parallel*** to the link *AB.*

We know that tangential component of the acceleration of *B* with respect to *A*,

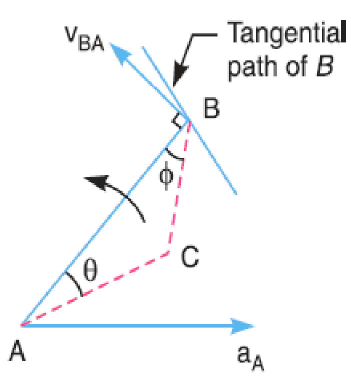
$$a^t_{AB} = \alpha \cdot Legth\ of\ link\ AB = \alpha \cdot AB$$

This tangential component of acceleration acts parallel to the velocity v_{BA}. In other words, it acts ***perpendicular*** to the link *AB.*

3.2 Acceleration of a Point on a Link

Consider two points A and B on the rigid link, as shown in Fig. (a). Let the acceleration of the point A i.e. a_A is known in magnitude and direction and the direction of path of B is given. The acceleration of the point B is determined in magnitude and direction by drawing the acceleration diagram as discussed below.

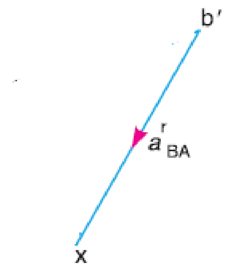
To Draw the acceleration, diagram the following steps to be follow:



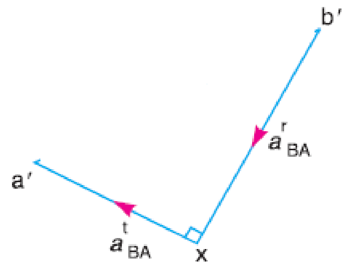
(*a*) Points on a Link.

1) From any point o' draw vector $o'a'$ // to the direction of absolute acceleration at point A (i.e aA) to some suitable scale.

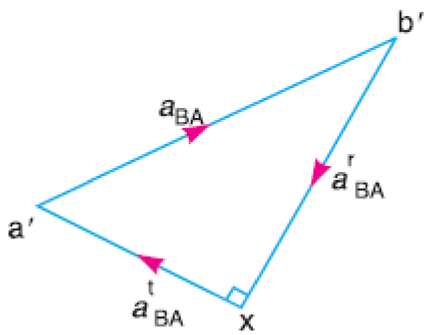
2) From point b´ draw vector $b'x$ // to AB represent the radial component of acceleration($a_B{}^r$) with known magnitude and direction.



3) From point x draw vector xa' $\perp$ AB to represent the tangential component of acceleration ($a_B{}^t$).



3) Join $b'a'$, the vector $b'a'$ known acceleration image of link AB or total acceleration of point B i.e (a_{AB}).



Note that:

a) Tangential component $\mathbf{a^t}$

$$a^t = r \cdot \propto \ and\ always \perp r$$

b) Radial component $\boldsymbol{a^r}$

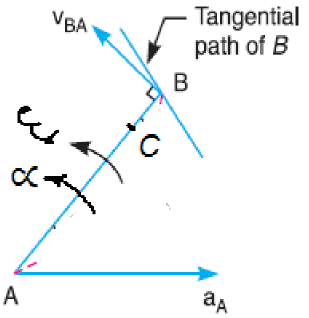
$$a^r = r.\omega^2 = v^2 \cdot r \text{ is always//r}$$

r.. length of link ***AB***

$\boldsymbol{a^r}$ is drawn first then at where at $\perp$ $\boldsymbol{a^r}$

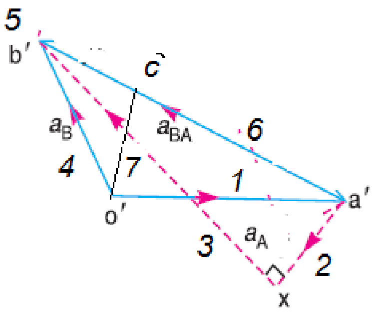
3.2 Acceleration diagram of a point on a link:

A two-point ***A***, ***B*** on link ***AB*** as shown in the figure below. Let the acceleration of point ***A*** i.e a_A is known in magnitude and direction, if the path of point ***B*** is known, to determine the acceleration of point ***B*** magnitude and direction the following procedure can be follows:



From any point *o'*, draw vector *o'a'* parallel to the direction of absolute acceleration at point *A i.e.* a_A , to some suitable scale, as

1) From any point ***o´*** draw vector $o´a´$ //to the direction of **absolute acceleration** at point ***A*** (i.e a_A) to some suitable scale.



2) Draw vector $a´$x//to AB which represent ($a_B{}^r$) and

$$a_B^r = v_B^2 \cdot AB = \omega_{AB}^2 \cdot AB$$

With the same scale.

3) From point ***x*** draw vector x$b´$ ⊥ to $a´$x

4 From ***o´*** draw line//to tangental path to **B**

5) Now the vectors **x*b´*** and ***o´b´*** will intersect at point ***b´*** , and the values of $\mathbf{a_B}$ and $a_B{}^t$ can be measured to the scale.

6) Join ***a´*** and ***b´***the acceleration image of link ***AB*** will represented by vector $a´b´$. 7) The acceleration of any point (***c***) on ***AB*** may be obtained by

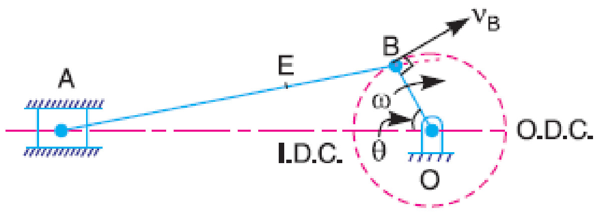
$$\frac{a´c}{a´b´} = \frac{AC}{AB} \xrightarrow{yields} a´c = a_{CA} \; and \; o´c = a_c$$

8) The value of angular acceleration ∝then can be obtained by

$$\alpha = \frac{a^t}{AB}$$

3.3 Acceleration in the Slider Crank Mechanism:

A slider crank mechanism shown in the figure below.



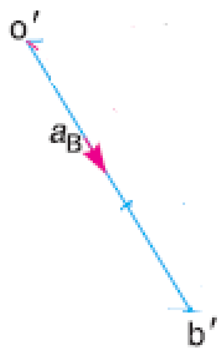
$$we\ know\ that\ v_B = \omega_B \cdot BO\ and, a_{BO}^r = \omega_{BO}^2 \cdot OB = \frac{v_{BO}^2}{OB}$$

The acceleration diagram can be drawn as:

1) Draw vector ***o´b´*** // to ***BO*** in magnitude of $a_B{}^r$, with suitable scale. since point ***B*** moves with a constant ***ω*** therefore will be no tangential component acceleration.

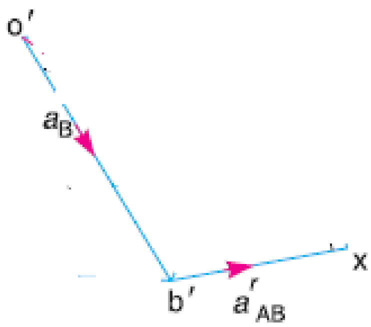
Note: A point at the end of a link which moves with constant angular velocity has no tangential component of acceleration.

$$\because \omega_B = const \xrightarrow{yields} a_B^t = 0 \xrightarrow{\text{yields}} a_B^r = a_B = \omega_{BO}^2 \cdot BO$$



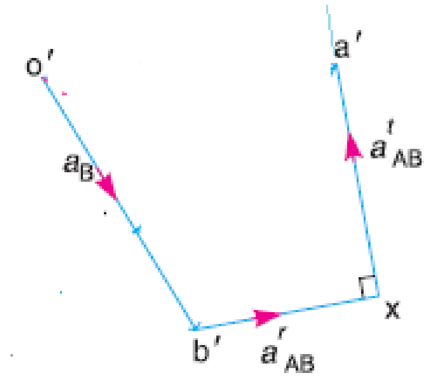
2) From point ***b'***, draw vector ***b'x*** parallel to ***BA***. The vector ***b'x*** represents the radial component of the acceleration of ***A*** with respect to ***B*** whose magnitude is given by:

$$a_{BA}^r = \omega_{BA}^2 \cdot AB = \frac{v_{BA}^2}{AB}$$



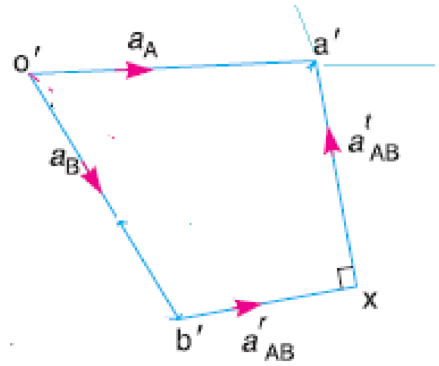
3) From point ***x***, draw vector ***xa′*** perpendicular to ***b′x*** (or ***AB***). The vector ***xa′*** represents the tangential component of the acceleration of ***A*** with respect to ***B*** *i.e.* a^r_{AB} .

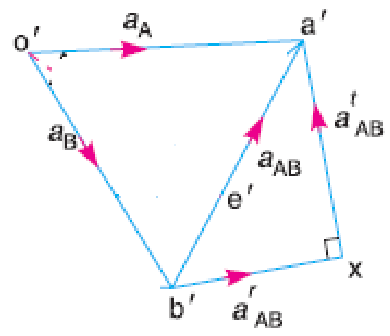
Note: When a point moves along a straight line, it has **no centripetal or radial** component of the acceleration.



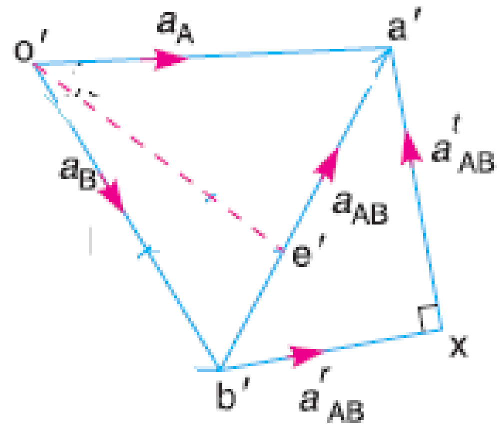
4) Since the point **A** reciprocates along **AO**, therefore the acceleration must be // to velocity. Draw from ***o´*a*´*** vector ***o´a´*** // to **AO**, will intersect the vector **x*a´*** at point ***a´***. Now the acceleration of the piston or the slider ***A*** (a_A) and a^t_{AB} may be measured to the scale.

5) The vector ***b′a′***, which is the sum of the vectors ***b′ x*** and ***x a′***, represents the total acceleration of *A* with respect to *B* *i.e.* a_{AB}. The vector ***b′a′*** represents the acceleration of the connecting rod ***AB***.





6) The acceleration of any other point on ***AB*** such as ***E*** may be obtained by dividing the vector ***b'a'*** at ***e'*** in the same ratio as ***E*** divides ***AB***



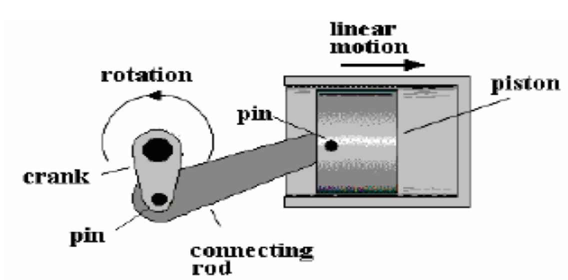
$$\frac{a'e'}{a'b'} = \frac{AE}{AB} \text{ or by measuring } o'e' \text{ to the scale}$$

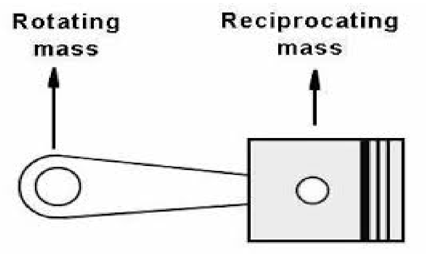
7) The angular acceleration of the connecting rod ***AB*** ($\propto_{AB}$) can be obtained from:

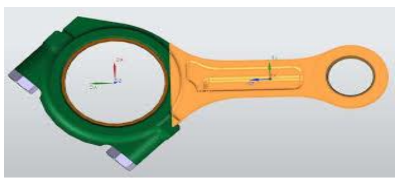
$$\alpha_{AB} = \frac{a_{AB}^{t}}{AB} \ (clockwise\ about\ B)$$

3.4. Reciprocating Machines

Reciprocating machines here means a piston reciprocating in a cylinder and connected to a crank shaft by a connecting rod. You can skip the derivation of the acceleration by going to the next page-First let s establish the relationship between crank angle, and the displacement, velocity and acceleration of the piston.

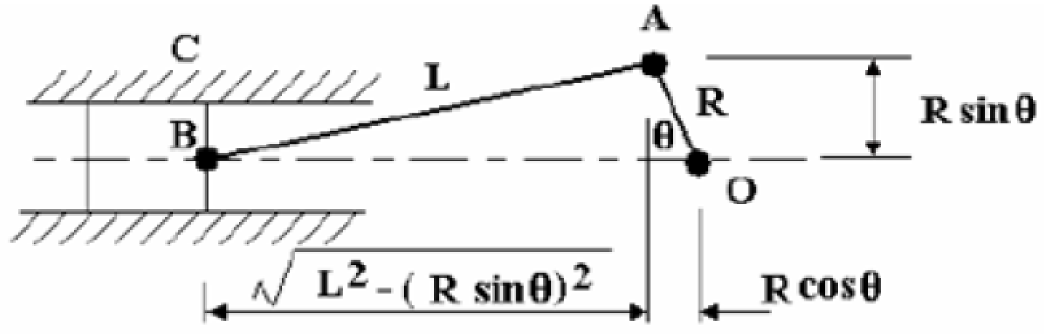






3.3.1 Derivation of acceleration equation:

A crank, connection rod, and piston mechanism as show in the figure.



When θ=0 the piston will be furthest left at a distance of:

$$L + R \text{ from point } O$$

Take this as reference point and measure displacement x from there. Remember that:

$$\theta = \omega \cdot t, and\ \omega = \frac{2 \cdot \pi \cdot N}{60}$$

The displacement is then:

$$x = (L + R) - [\sqrt{L^2 - (R \cdot \sin\theta)^2} + R \cdot \cos\theta]; put\ n = \frac{L}{R} \xrightarrow{yields} x$$

$$= R \cdot [(n-1) - \cos\theta - \sqrt{n^2 - (\sin\theta)^2}]$$

Differentiate to get the velocity:

$$v = \frac{dx}{dt} \quad = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$$

$$v = \omega \times \frac{dx}{d\theta}$$

$$\therefore R\sin\theta = L$$

$$\sin\theta^2 = 2\sin\theta\cos\theta$$

$$v = \frac{dx}{dt} = \omega \cdot \frac{dx}{d\theta} = \omega \times R \frac{d\left[(n+1) - \cos\theta - (n^2 - \sin\theta^2)^{\frac{1}{2}}\right]}{d\theta}$$

$$= \omega \cdot R\left[0 + \sin\theta - \frac{1}{2}(n^2 - sin\,\theta^2)^{\frac{-1}{2}} \cdot (-2\sin\theta\cos\theta)\right]$$

$$v = \omega \times R\left[\sin\theta - \left(\frac{-2\sin\theta\cos\theta}{2(n^2 - \sin\theta^2)^{\frac{1}{2}}}\right)\right] \qquad \because 2\sin\theta\cos\theta = \sin 2\theta$$

$$v = \omega \times R\left[\sin\theta + \frac{\sin 2\theta}{2(n^2 - \sin\theta^2)^{\frac{1}{2}}}\right]$$

Differentiate again and simplify to get acceleration:

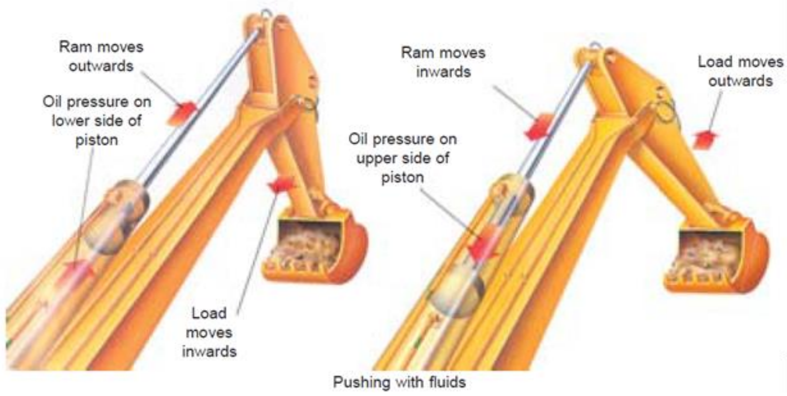
$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} \qquad and \qquad \omega = \frac{d\theta}{dt}$$

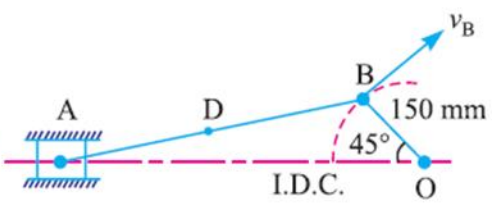
$$\therefore a = \omega \times \frac{dv}{d\theta}$$

$$a = \omega^2 \cdot R[\cos\theta + \frac{sin^2(2\cdot\theta)}{4\cdot\left(n^2 - sin^2(\theta)\right)^{\frac{3}{2}}} + \frac{\cos\left(2\cdot\theta\right)}{\left(n^2 - sin^2(\theta)\right)^{\frac{1}{2}}}$$

EX:1 The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine:

1. linear velocity and acceleration of the midpoint of the connecting rod,
2 angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead center position.

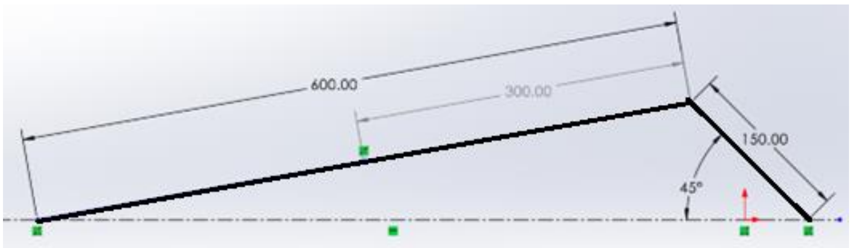




Solution.
Given: n_{BO} = 300 r.p.m. or ω_{BO} = 2 π × 300/60 = 31.42 rad/s; ***OB*** = 150 mm = 0.15 m; ***BA*** = 600 mm = 0.6 m.
We know that linear velocity of ***B*** with respect to ***O*** or velocity of *B*,

$$v_{BO} = v_B = \omega_{BO} \cdot OB = 31.42 \cdot 0.15 = 4.713\ m/s$$

1. Linear velocity of the midpoint of the connecting rod
First of all, draw the space diagram, to some suitable scale(1:2); as shown in Figure below



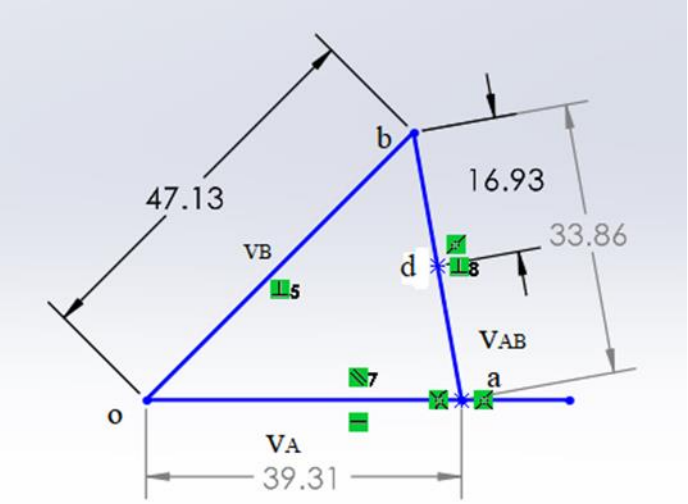
Spece diagram with scale (1:2)
Now the velocity diagram, as shown in Figure below, is drawn as discussed below:

1**)** Draw vector ***ob*** perpendicular to ***BO***, to some suitable scale(10mm:1m/sec), to represent the velocity of ***B*** with respect to ***O*** or simply velocity of ***B*** *i.e.* v_{BO} or v_B, such that:

vector *ob* = v_{BO} = v_B = 4.713 m/s

$$\overline{v_b} = 4.713 \cdot 10 = 47.13\,mm$$

2) From point *b*, draw vector ***ba*** perpendicular to ***BA*** to represent the velocity of ***A*** with respect to ***B*** *i.e.* v_{AB} , and from point ***o*** draw vector ***oa*** parallel to the motion of ***A*** (which is along ***AO***) to represent the velocity of ***A*** *i.e.* v_A. The vectors ***ba*** and ***oa*** intersect at ***a***.



By measurement, we find that velocity of ***A*** with respect to ***B***,

$$v_{AB} = vector\ \overline{ba} = 33.86/10 = 3.386\,m/sec$$

$$Velocity\ of\ A, v_A = Vector\ oa = \frac{39.31}{10} = 3.931\,m/sec$$

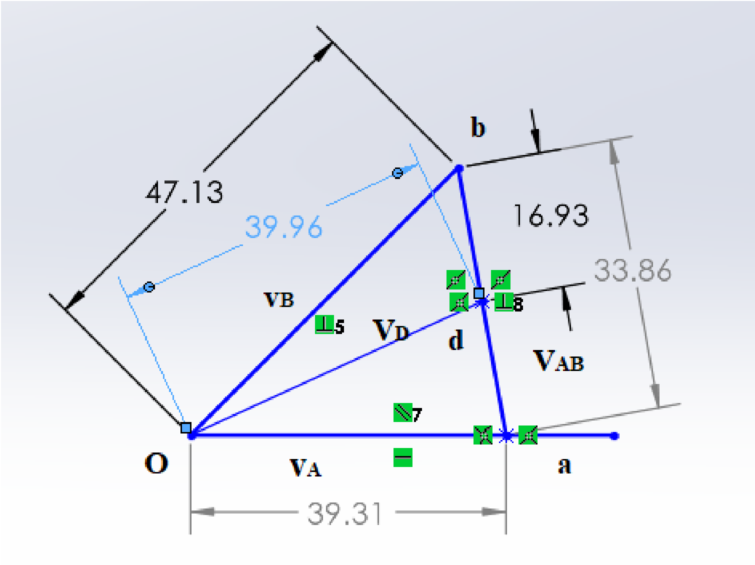
3) In order to find the velocity of the midpoint ***D*** of the connecting rod ***AB***, divide the vector ***ba*** at ***d*** in the same ratio as ***D*** divides ***AB***, in the space diagram. In other words,

$$\frac{bd}{ba} = \frac{BD}{BA} \xrightarrow{yields} \frac{bd}{33.86} = \frac{300}{600} \xrightarrow{yields} bd = 16.93\,mm$$

$$bd = 16.93\,mm$$

Note: Since ***D*** is the midpoint of ***AB***, therefore ***d*** is also midpoint of vector ***ba***. Join ***od***. Now the vector ***od*** represents the velocity of the midpoint ***D*** of the connecting rod *i.e.* v_D.. By measurement, we find that:

v_D = vector ***od*** = 39.96/10=3.996 *m/s* **Ans.**



Acceleration of the midpoint of the connecting rod

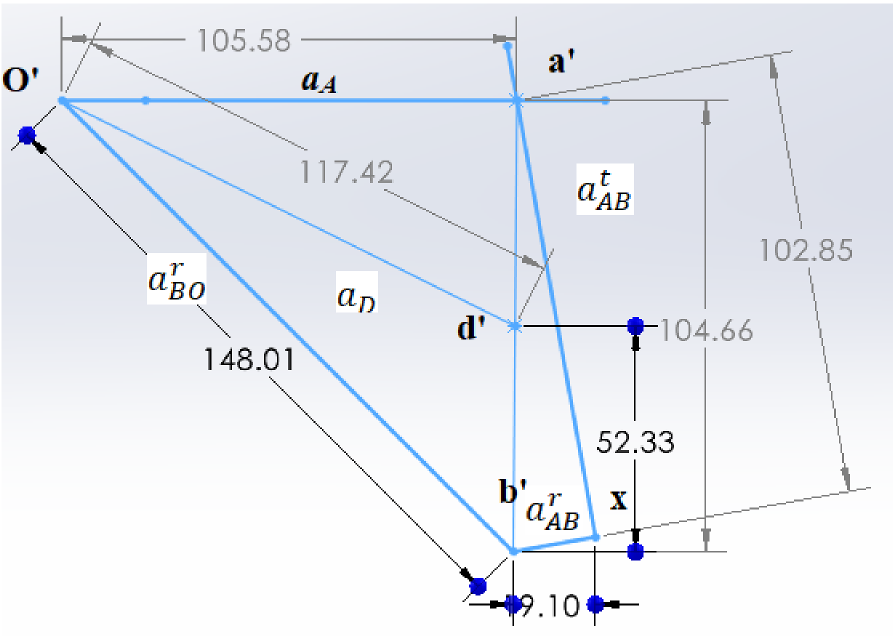
We know that the radial component of the acceleration of ***B*** with respect to ***O*** or the acceleration of ***B***,

$$a_{BO}^{r} = a_B = \frac{v_B^2}{OB} = \frac{4.713^2}{0.15} = 148.01\ m/s^2$$

and the radial component of the acceleration of ***A*** with respect to ***B***,

$$a_{AB}^{r} = \frac{v_{BA}^2}{BA} = \frac{3.386^2}{0.6} = 19.1\ m/s^2$$

Now the acceleration diagram, as shown in figure below is drawn as discussed below:



Acceleration diagram with scale 1mm:1m/s^2

1) Draw vector ***o'b'*** parallel to ***BO***, to some suitable scale(1mm:1m/s^2), to represent the radial component of the acceleration of ***B*** with respect to ***O*** or simply acceleration of ***B*** *i.e.* a^r_{BO} or a_B, such that

$$a^r_{BO} = 148.01\, m/s^2$$

Note: Since the crank *OB* rotates at a constant speed, therefore there will be no tangential component of the acceleration of *B* with respect to *O*.

2) The acceleration of ***A*** with respect to ***B*** has the following two components:

(*a*) The radial component of the acceleration of ***A*** with respect to ***B*** *i.e.* a^r_{AB}, and

(*b*) The tangential component of the acceleration of ***A*** with respect to ***B*** *i.e.* a^t_{AB}. These two components are mutually perpendicular.

Therefore, from point ***b'***, draw vector ***b'x*** parallel to ***AB*** to represent a^r_{AB}= 19.1m/s^2 and from point *x* draw vector perpendicular to vector *b' x* whose magnitude is yet unknown.

3. Now from ***o'***, draw vector ***o'a'*** parallel to the path of motion of ***A*** (which is along ***AO***) to represent the acceleration of ***A*** *i.e.* a_A . The vectors in point 2 and previous vector intersect at ***a'***. Join ***a'b'***, a_A=105.58 m/s^2.

4. In order to find the acceleration of the midpoint ***D*** of the connecting rod ***AB***, divide the vector ***a'b'*** at ***d'*** in the same ratio as ***D*** divides ***AB***. In other words

$$\frac{b'd'}{b'a'} = \frac{BD}{BA} \xrightarrow{yields} \frac{b'd'}{104.66} = \frac{300}{600} \xrightarrow{yields} b'd' = 52.33mm$$

Note: Since *D* is the midpoint of *AB*, therefore *d'* is also midpoint of vector *b' a'*.

5) Join ***o'd'***. The vector ***o'd'*** represents the acceleration of midpoint ***D*** of the connecting rod *i.e.* a_D. By measurement, we find that

$$a_D = vector\ o'd' = \frac{117.42}{1} = 117.42\, m/s^2$$

2. ***Angular velocity of the connecting rod***

We know that angular velocity of the connecting rod *AB*,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.386}{0.6} = 5.6\, rad/s\ (anticlockwise\ about\ B).$$

3. Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a^t_{AB} = vector\ xa' = 102.85/1 = 102.85\, m/s^2$$

We know that angular acceleration of the connecting rod *AB*,

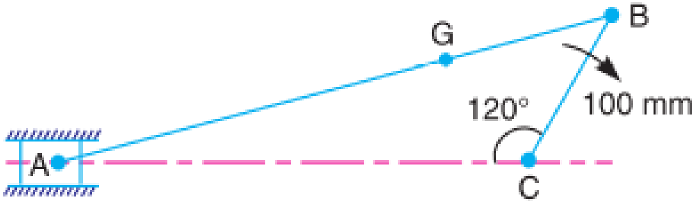
$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{102.85}{0.6} = 171.4 rad/s^2 (cw\ about\ B)$$

Example2

An engine mechanism is shown in Figure below. The crank CB = 100 mm and the connecting rod BA = 300 mm with center of gravity G, 100 mm from B. In the position shown, the crankshaft has a speed of 75 rad/s and an angular acceleration of 1200 rad/s^2. Find:

1. velocity of G and angular velocity of AB,

2. acceleration of G and angular acceleration of AB.



Solution.

Given: ω_{BC} = 75 rad/s ; α_{BC} = 1200 rad/s^2, CB = 100 mm = 0.1 m; BA = 300 mm.

We know that velocity of B with respect to C or velocity of B,

$$v_{BC} = v_B = \omega_{BC} \cdot c_B = 75 \cdot 0.1 = 7.5 \frac{m}{sec} \perp BC$$

Since the angular acceleration of the crankshaft, α_{BC} = 1200 rad/s$_2$, therefore tangential component of the acceleration of B with respect to C is:

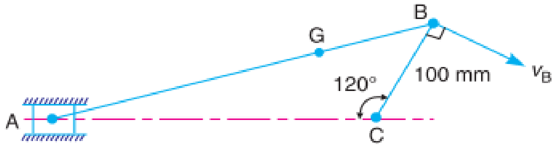
$$a_{BC}^t = \alpha_{BC} \cdot CB = 1200 \cdot 0.1 = 120\, m/s^2 \perp BC$$

Note: When the angular acceleration is not given, then there will be no tangential component of the acceleration.

Velocity of G and angular velocity of AB

First of all, draw the space diagram, to some suitable scale, as shown in Fig. a

$$S = \frac{DS}{AS} = \frac{50\, mm}{300mm} = \frac{1}{6}$$



Space diagram with scale 1/6

Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below:

1) Draw vector ***cb*** ⊥ ***CB***, to some suitable scale(***1m/sec:1cm***), to represent the velocity of ***B*** with respect to ***C*** or velocity of ***B*** (i.e., $\boldsymbol{v_{BC}}$ or $\boldsymbol{v_B}$), such that

$$v_B = 7.5\frac{m}{sec}\cdot 10 = 75\ mm \perp BC$$

2) . From point b, draw vector ⊥B A to represent the velocity of ***A*** with respect to ***B*** i.e. $\boldsymbol{v_{AB}}$, and from point ***c***, draw vector parallel to the path of motion of ***A*** (which is along ***AC***) to represent the velocity of ***A*** i.e. $\boldsymbol{v_A}$. both vectors intersect at ***a.***

From velocity diagram, we find that velocity of ***A*** with respect to ***B,***

a)***$\boldsymbol{v_{AB}}$=39.15 cm 1/10=3.915 m/sec***

We know that angular velocity of ***AB,***

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{3.915}{0.3} = 13.05(clockwise\ about\ A)$$

b)We know that angular velocity of slider **A,**

$\boldsymbol{v_A}$=53.69 cm 1/10=5.369 m/sec

to fine velocity of G there are two solutions:

a)

$$v_{GA} = \omega_{GA}\cdot GA = \omega_{AB}\cdot GA = 13.05\cdot 0.2 = 2.61m/sec$$

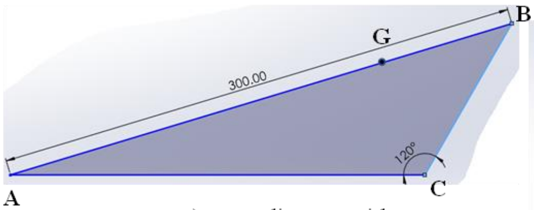
b)Since the point ***G*** lies on ***AB***, therefore divide vector ***ab*** at ***g*** in the same ratio as ***G*** divides ***AB*** in the space diagram. In other words,

$$\frac{\overrightarrow{ag}}{\overrightarrow{ab}} = \frac{AG}{AB} \xrightarrow{yields} \frac{\overrightarrow{ag}}{3.915cm} = \frac{200}{300}$$

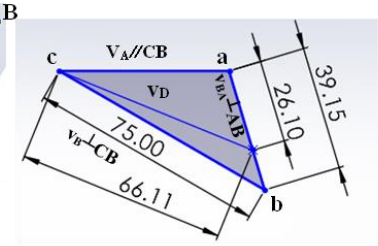
$$ag = 26.1\ mm$$

The vector ***cg*** represents the velocity of **G**. By measurement, we find that velocity of ***G***,

$$v_{GA} = 26.1mm\cdot\frac{1}{10} = 2.61m/sec$$



a) space diagram with scale 1/6



b) velocity diagram with scale 1/1

Acceleration of G and angular acceleration of AB

a) We know that radial component of the acceleration of ***B*** with respect to ***C***,

$$a_{BC}^{r} = \frac{v_{BC}^{2}}{BC} = \frac{7.5^{2}}{0.1} = 562.5m/s^{2}$$

and radial component of the acceleration of ***A*** with respect to ***B***

$$a_{AB}^{r} = \frac{v_{AB}^{2}}{AB} = \frac{3.915^{2}}{0.3} = 51.1m/s^{2}$$

Now the acceleration diagram, as shown in Fig.(c), is drawn as discussed below:

a) Draw vector c' b'' parallel to ***CB***, to some suitable scale (1:10), to represent the radial component of the acceleration of ***B*** with respect to ***C,*** i.e.a^{r}_{BC}, such that

$$vector c'b'' = a_{BC}^{r} = 562.5m/s^{2}$$

b) From point ***b''***, draw vector ⊥ c' b' or CB to represent the tangential component of the acceleration of ***B*** with respect to ***C*** i.e. a^{t}_{BC}, such that

$$vector b''b' = a_{BC}^{t} = 120\ m/s^{2}$$

c) Join ***c'b'***. The vector ***c'b'*** represents the total acceleration of ***B*** with respect to ***C*** i.e., ***c'b'***=57.52 mm

$$a_{BC} = 57.52 \cdot 10 = 575.2\ m/s^{2}$$

d) . From point ***b'***, draw vector ***b'x*** parallel to ***BA*** to represent radial component of the acceleration of ***A*** with respect to ***B*** i.e. a^{r}_{AB} such that

$$vector b'x = a_{AB}^{r} = 51.1m/s^{2}$$

e) From point ***x***, draw vector perpendicular to vector ***b'x*** or ***BA*** to represent tangential component of the acceleration of ***A*** with respect to ***B*** i.e. a^t_{AB}, whose magnitude is not yet known.

f) Now draw vector from point ***c'*** parallel to the path of motion of ***A*** (which is along ***AC***) to represent the acceleration of ***A*** i.e. a_A. The both vectors intersect at ***a'.*** Join ***b'a'***. The vector ***b'a'*** represents the acceleration of ***A*** with respect to ***B*** i.e., a_{AB}.

g) In order to find the acceleration of ***G***, divide vector ***a'b'*** in ***g'*** in the same ratio as ***G*** divides ***BA*** in Fig.(a). Join ***c'g'.*** The vector ***c'g'*** represents the acceleration of ***G***

$$\frac{\overrightarrow{a'g'}}{\overrightarrow{a'b'}} = \frac{AG}{AB} \xrightarrow{yields} \frac{\overrightarrow{a'g'}}{49.59mm} = \frac{200}{300}$$

$$a'g' = 33.06mm$$

By measurement, we find that acceleration of ***G***

$$a_G = vector\ c'g' = 39.33 \cdot 10 = 393.3m/s^2$$

h) From acceleration diagram, we find that tangential component of the acceleration of ***A*** with respect to ***B***,

$$vector a'x = a^t_{AB} = 55.59 \cdot 10 = 555.9m/s^2$$

$$vector a'b' = a_{AB} = 55.83 \cdot 10 = 558.3m/s^2$$

∴ Angular acceleration of ***AB***,

$$\alpha_{AB} = \frac{a^t_{AB}}{AB} = \frac{555.9}{0.3} = 1853 rad/sec^2 \text{(Clockwise)}$$

EX 3

PQRS is a four-bar chain with link **PS** horizontal fixed. the lengths of the links are **PQ** - 62.5 mm; **QR**- 175mm; **RS**- 112.5 mm; and **PS** =200mm.The crank **PQ** rotates at 10 rad/s clockwise. Draw:

1) the velocity and acceleration diagram of links QR and RS when angle **QPS** =60^o and **Q** and **R** lie on the same side of **PS**.

2)Find the angular velocity and angular acceleration of links **QR** and **RS**

Solution.

Given:

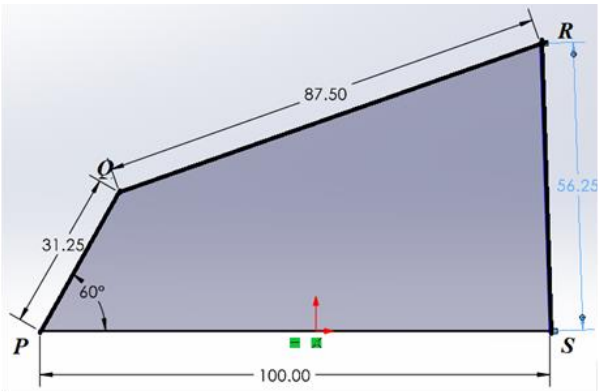
ω_{QP}= 10 rad/s. **PQ** = 62.5mm = 0.0625 m; **QR** = 175 mm = 0.175 m; **RS**= 112.5 mm = 0.1125 m; **PS** - 200 mm = 0 2 m

We know that velocity of **Q** with respect to **P** or v_{QP}

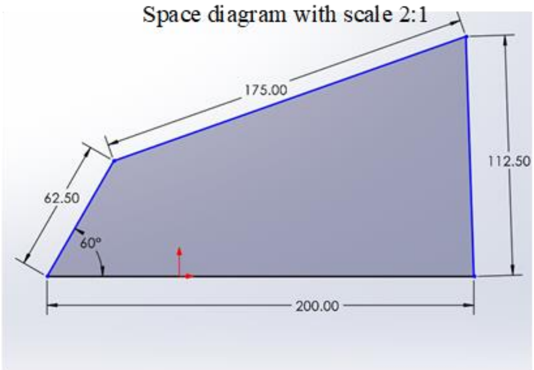
$$v_{QP} = v_Q = \omega_{QP} \cdot PQ = 10 \cdot 0.0625 = 0.625\, m/s \perp \text{PQ}$$

Angular velocity of links QR and RS

First of all, draw the space diagram of a four-bar chain, to some suitable scale(1:2), as shown in figure below (a). Now the velocity diagram as shown in figure below,



Space diagram with scale 2:1



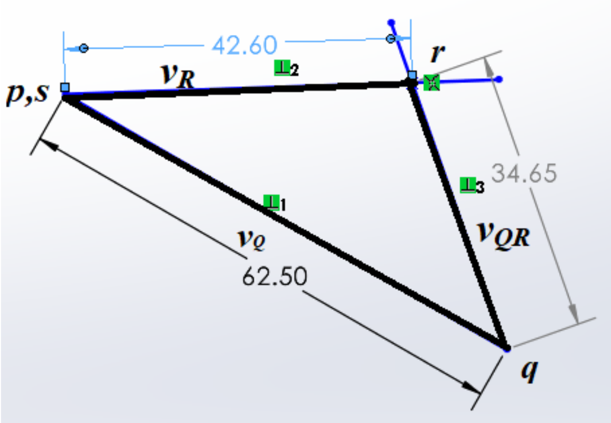
Actual size

is drawn as discussed below:

1. Since ***P*** and ***S*** are fixed points, therefore these points lie at one place in velocity diagram. Draw vector ***pq*** perpendicular to ***PQ***, to some suitable

scale(100mm:1m/sec), to represent the velocity of ***Q*** with respect to ***P*** or velocity of ***Q*** *i.e.* $\boldsymbol{v_{QP}}$ or $\boldsymbol{v_Q}$ such that

$$vector\ pq = v_{QP} = v_Q = 0.625 \xrightarrow{yields} \bar{v}_Q = 0.625 \cdot 100 = 62.5\ mm$$



Velocity diagram with scale 100 mm;1m/sec

2. From point ***q***, draw vector ***qr*** perpendicular to ***QR*** to represent the velocity of ***R*** with respect to ***Q*** (*i.e.* $\boldsymbol{v_{RQ}}$) and from point ***s***, draw vector ***sr*** perpendicular to ***SR*** to represent the velocity of ***R*** with respect to ***S*** or velocity of ***R*** (*i.e.* $\boldsymbol{v_{RS}}$ or $\boldsymbol{v_R}$). The vectors ***qr*** and ***sr*** intersect at ***r***. By measurement, we find that

$$v_{RQ} = vector\ qr = \frac{34.65}{100} = \frac{0.3465m}{sec}, and v_{RS} = v_R = \frac{42.6}{100} = 0.426\ m/sec$$

We know that angular velocity of link ***QR***,

$$\omega_{QR} = \frac{v_{QR}}{QR} = \frac{0.3465}{0.175} = 1.98\ rad/s\ (anticlockwise\ about\ Q).$$

and angular velocity of link *RS*,

$$\omega_{RS} = \frac{v_{RS}}{RS} = \frac{0.426}{0.1125} = 3.78\ rad/s\ (clockwise\ about\ R).$$

Angular acceleration of links QR and RS

Since the angular acceleration of the crank ***PQ*** is not given, therefore there will be no tangential component of the acceleration of ***Q*** with respect to ***P***.

We know that radial component of the acceleration of ***Q*** with respect to ***P*** (or the acceleration of ***Q***),

$$a_{QP}^r = a_Q = \frac{v_Q^2}{PQ} = \frac{0.625^2}{0.0625} = 6.25\ m/s^2$$

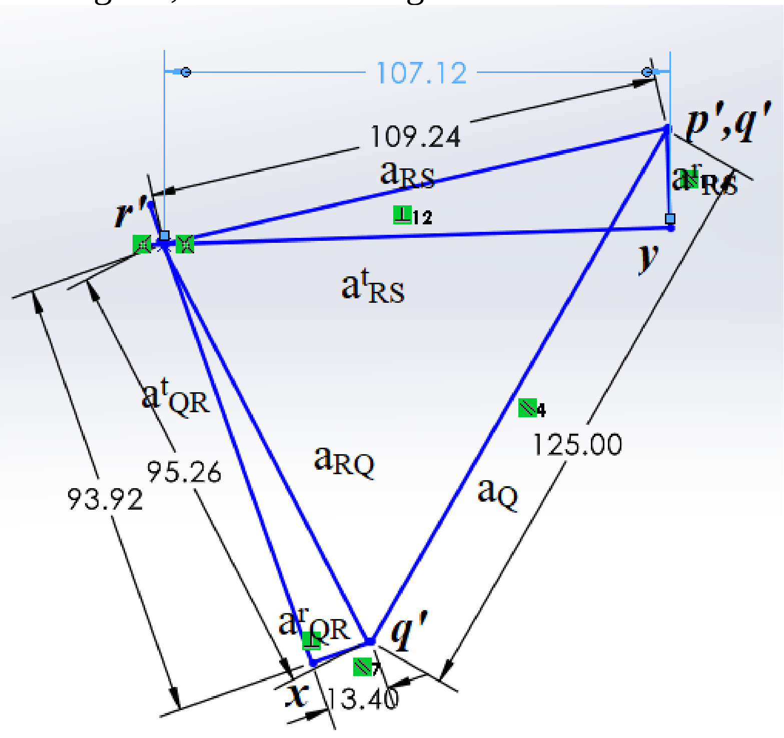
Radial component of the acceleration of *R* with respect to *Q*,

$$a_{RQ}^r = \frac{v_{RQ}^2}{QR} = \frac{0.3465^2}{0.175} = 0.67\ m/s^2$$

and radial component of the acceleration of *R* with respect to *S* (or the acceleration of *R*),

$$a_{RS}^r = a_R^r = \frac{v_{RS}^2}{RS} = \frac{0.426^2}{0.1125} = 1.61\, m/s^2$$

The acceleration diagram, as shown in Figure below



Acceleration diagram

is drawn as follows:

1. Since ***P*** and ***S*** are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector ***p'q'*** parallel to ***PQ***, to some suitable scale(20mm/1 m/s^2), to represent the radial component of acceleration of ***Q*** with respect to ***P*** or acceleration of ***Q*** *i.e* $\boldsymbol{a}^r_{\mathbf{QP}}$ or $\boldsymbol{a}_{\mathbf{Q}}$ such that

$$a_{QP}^r = a_Q = 6.26\frac{m}{s2} \xrightarrow{yields} vetor\ p'q' = \bar{a}_Q = 6.26 \cdot 20 = 125mm$$

2. From point ***q'***, draw vector ***q'x*** parallel to ***QR*** to represent the radial component of acceleration of ***R*** with respect to ***Q*** *i.e.* $\boldsymbol{a}^r_{\boldsymbol{RQ}}$ such that

$$a_{RQ}^r = 0.67m/s2 \xrightarrow{yields} vector\ q'x = \bar{a}_{RQ} = 0.67 \cdot 20 = 13.4mm$$

3. From point ***x***, draw vector perpendicular to ***QR*** to represent the tangential component of acceleration of ***R*** with respect to ***Q*** *i.e* **RQ** *at* whose magnitude is not yet known.

4. Now from point ***s'***, draw vector ***s'y*** parallel to ***SR*** to represent the radial component of the acceleration of ***R*** with respect to ***S*** *i.e.* $\boldsymbol{a}^r_{\mathbf{RS}}$ such that

$$a_{RS}^r = 1.61m/s2 \xrightarrow{yields} vector\ s'y = \bar{a}_{RS} = 1.61 \cdot 20 = 32.2mm$$

5. From point *y*, draw vector perpendicular to ***SR*** to represent the tangential component of acceleration of ***R*** with respect to ***S*** *i.e.* a^t_{RS}

6. The vectors perpendicular vector in 4 and 5 intersect at ***r'***. Join ***p'r'*** and ***q'r'***. By measurement, we find that

$$a^t_{RQ} = vector\ xr' = \frac{93.92}{20} = 4.696\frac{m}{s^2}, and a^t_{RS} = vector\ yr' = \frac{107.12}{20} = 5.356\frac{m}{s^2}$$

We know that angular acceleration of link ***QR***,

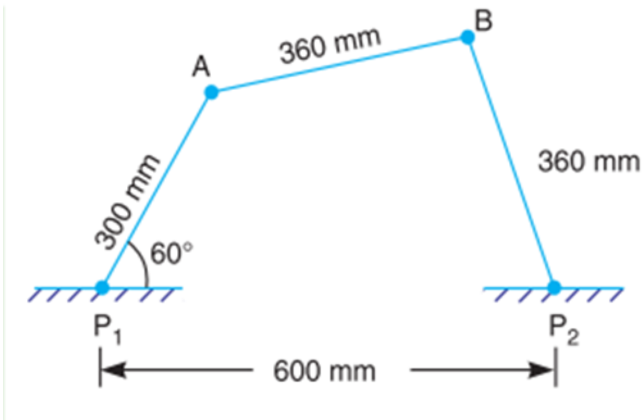
$$\alpha_{QR} = \frac{a^t_{QR}}{QR} = \frac{4.696}{0.175} = 26.83 rad/s^2 (anticw\ about\ R$$

and angular acceleration of link *RS*,

$$\alpha_{RS} = \frac{a^t_{RS}}{BA} = \frac{5.356}{0.1125} = 47.6 rad/s^2 (anticw\ about\ R$$

Example 5

The dimensions and configuration of the four-bar mechanism, shown in Fig. below, are as follows: $P_1A = 300$ mm; $P_2B = 360$ mm; AB = 360 mm, and $P_1P_2 = 600$ mm The angle $AP_1P_2 = 60°$. The crank P_1A has an angular velocity of 10 rad/s and an angular acceleration of 30 rad/s^2, both clockwise. Determine the angular velocities and angular accelerations of P_2B, and AB and the velocity and acceleration of the joint B.



Solution. Given: $\omega_{AP1} = 10$ rad/s; $\alpha_{AP1} = 30$ rad/s^2; $P_1A = 300$ mm = 0.3 m ; P_2B = A B = 360 mm = 0.36 m

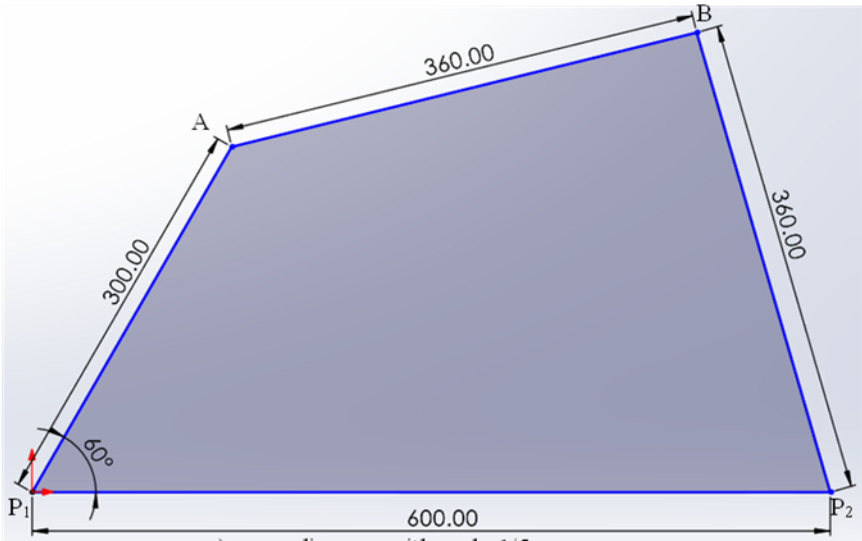
We know that the velocity of ***A*** with respect to ***P_1*** or velocity of ***A***,

$$v_{AP_1} = \omega_{AP_1} \cdot AP_1 = 10 \cdot 0.3 = 3\, m/s\,ec$$

Velocity of B and angular velocities of P_2B and AB

First of all, draw the space diagram, to some suitable scale, as shown in Fig. (a).

$$S = \frac{DS}{AS} = \frac{120\, mm}{360mm} = \frac{1}{5}$$



a) space diagram with scale 1/5

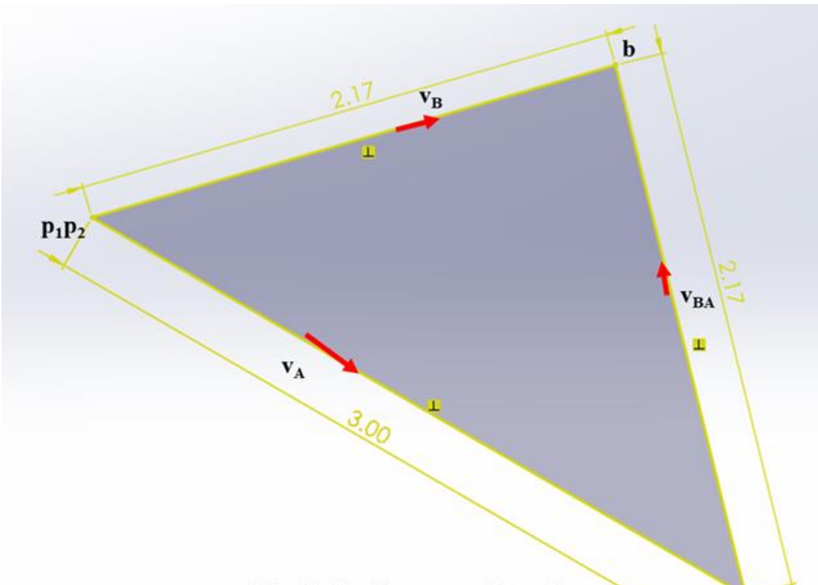
Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below:

to represent the velocity of $\boldsymbol{A}$ with respect to $\boldsymbol{P_1}$ or velocity of $\boldsymbol{A}$ i.e. $\boldsymbol{v_{AP1}}$ or $\boldsymbol{v_A}$, such that:

$$\text{vector } \boldsymbol{p_1 a} = \boldsymbol{v_{A\,P1}} = \boldsymbol{v_A} = 3 \text{ m/s}$$

$$S = \frac{DS}{AS} = \frac{12}{3} = \frac{4}{1}$$

Since $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ are fixed points, therefore these points lie at one place in velocity diagram. Draw vector $\boldsymbol{p_1 a}$ perpendicular to $\boldsymbol{P_1 A}$, to some suitable scale



b) velocity diagram with scale 1/1

By measurement, we find that:

v_B = vector p_2b = 2.17 m/s Ans.

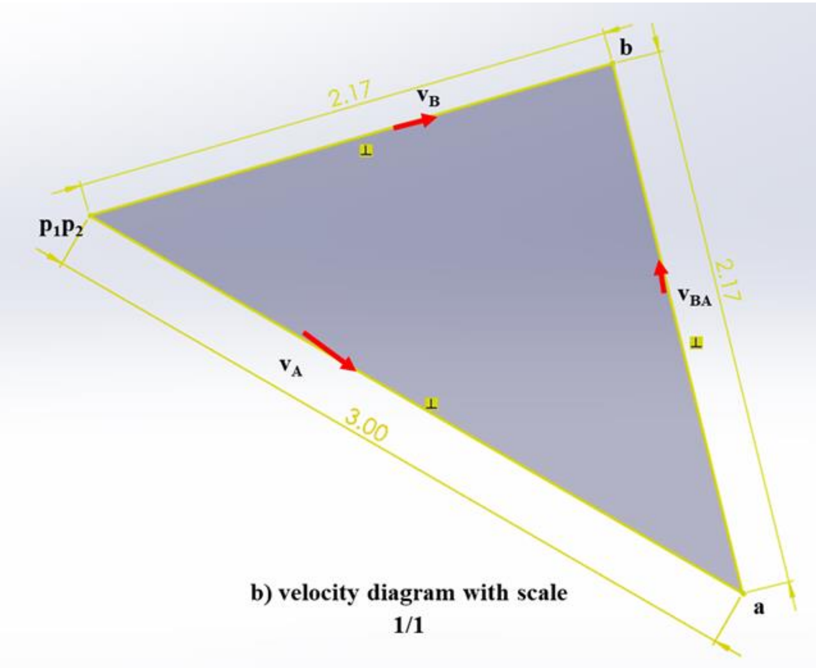
v_{BA} = vector ab = 2.17 m/s

We know that angular velocity of P_2B,

$$\omega_{P_2B} = \frac{v_{P_2B}}{P_2B} = \frac{2.17}{0.36} = 6.03\ rad/\sec\ ((\text{Clockwise})$$

and angular velocity of AB,

$$\omega_{AB} = \frac{v_{AB}}{AB} = \frac{2.17}{0.36} = 6.03\ rad/\sec\ (anti\text{clockwise})$$



b) velocity diagram with scale 1/1

Acceleration of B and angular acceleration of P2B and AB

We know that tangential component of the acceleration of ***A*** with respect to $\boldsymbol{P_1}$,

$$a^t_{AP_1} = \alpha_{AP_1} \cdot AP_1 = 30 \cdot 0.3 = 9\ m/s^2$$

Radial component of the acceleration of ***A*** with respect to $\boldsymbol{P_1}$,

$$a^r_{AP_1} = \frac{v^2_{AP_1}}{AP_1} = \frac{3^2}{0.3} = 30 m/s^2$$

Radial component of the acceleration of ***B*** with respect to ***A***.

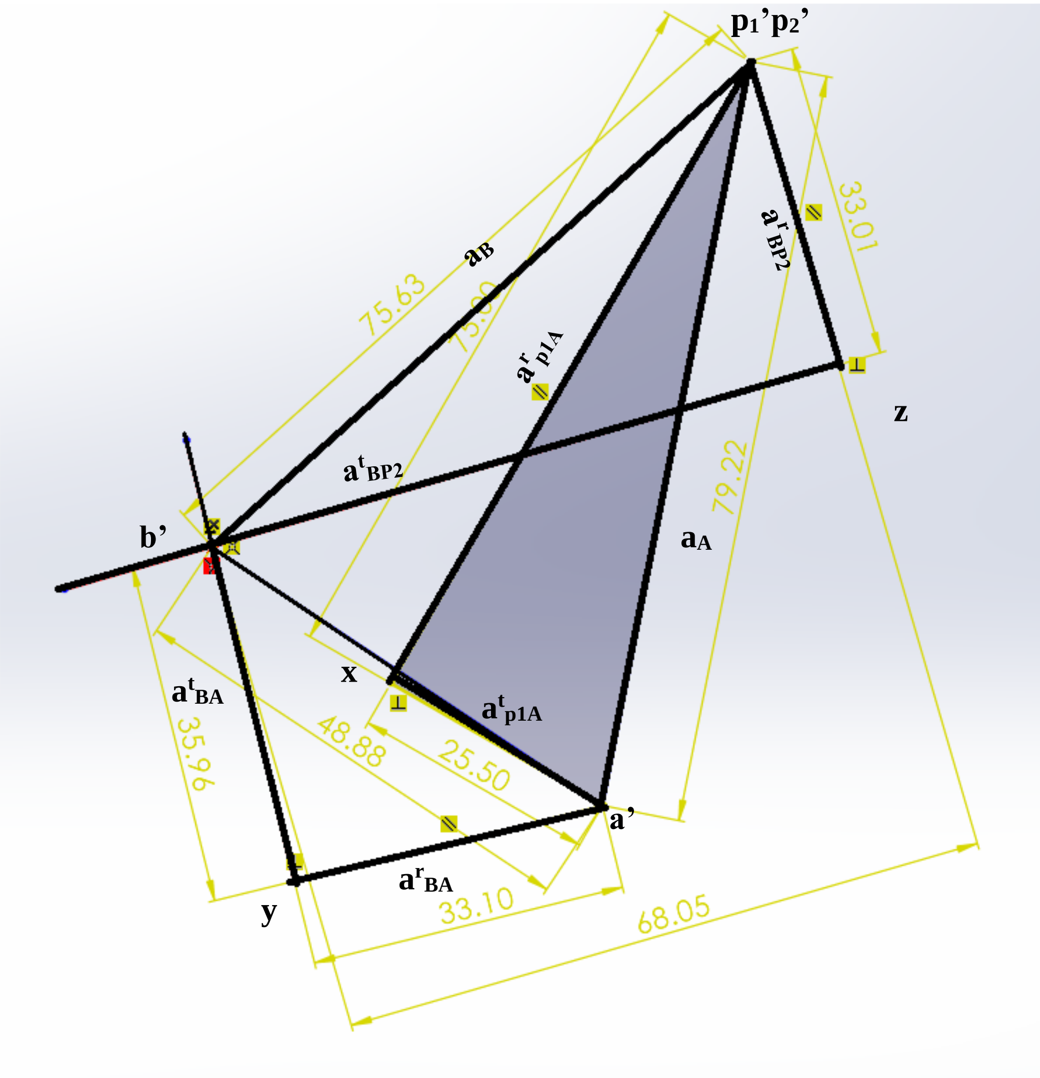
$$a_{AB}^r = \frac{v_{AB}^2}{AB} = \frac{2.17^2}{0.36} = 13.08 m/s^2$$

and radial component of the acceleration of ***B*** with respect to $\boldsymbol{P_2}$,

$$a_{BP2}^r = \frac{v_{BP2}^2}{BP2} = \frac{2.17^2}{0.36} = 13.08 m/s^2$$

The acceleration diagram, as shown in Fig. (c), is drawn as follows:

By measurement, we find that:



$$a_{BP_2} = a_B = vector \frac{p_2'b'}{0.25} = \frac{7.563}{0.25} = 30.252 m/s^2 \cdot$$

$$vector \frac{yb'}{0.25} = a_{BA}^t = \frac{3.596}{0.25} = 14.384\ m/s^2$$

$$vector\frac{zb'}{0.25} = a^t_{BP2} = \frac{6.805}{0.25} = 27.22 m/s^2$$

We know that angular acceleration of $\boldsymbol{P_2B}$,

$$\alpha_{BP2} = \frac{a^t_{BP2}}{BP2} = \frac{27.22}{0.36} = 75.61 m/s^2$$

and angular acceleration of ***AB***

$$\alpha_{AB} = \frac{a^t_{AB}}{AB} = \frac{14.384}{0.36} = 39.95\ m/s^2$$