5 Belts, ropes and chain drives.

Pulley drive systems depend on friction to enable the belt to grip the wheel and pull it around with it. To enable this, the belt must be tensioned, even when the wheels are stationary. This is unlike positive chain drive systems where teeth mesh with the chain and slip is not possible so no initial tension is required. Pulley drives are most often used to produce speed reduction between a motor and the machine being driven (e.g. a motor driving an air compressor). Many other applications exist from small rubber band drives in video recorders to large multi belt systems on heavy industrial equipment. On many modem systems, toothed belts are used (e.g. timing belt on a car engine) to prevent the belt slipping. This tutorial is only concerned with smooth belts.

Advantage belts used

a) They are simple and economical.

b) They can transmit Power over a considerable distance.

c) They can protect the machine from overloading by slipping of the belt over a pulley.

d) Belt drive can absorb shock and damp vibration.

Disadvantage of belt drive

- Limited Speed range.
- They are not compact.
- Considerable power loss.
- Short service life compared to other mode power transmission.
- The velocity ratio may vary due to belt slip.
- They inflict a heavy load on shafts and bearings.

5.1 Material used for Belts

a) Leather belts.

The most important material for the belt is leather. The best leather belts are made from 1.2 meters to 1.5 meters.

b) Cotton or fabric belts.

Most of the fabric belts are made by folding canvass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together.

c)Rubber belt.

The rubber belts are made of layers of fabric impregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease.

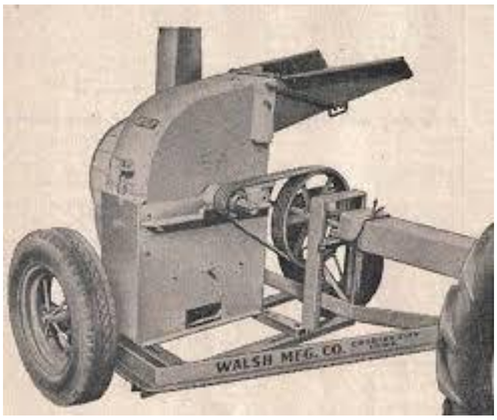
d)Balata belts.

These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not affected by animal oils or alkalizes. The strength of balata belts is 25 per cent higher than rubber belts.

5.2 Types of Belt Drives

- *The belt drives are usually classified into the following three groups depending on the amount of load transferred:*

1. ***Light drives***. These are used to transmit small powers at belt speeds up to about 10 m/s, as in agricultural machines and small machine tools.



Agricultural machines

2. ***Medium drives***. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

Machine tools

3. *Heavy drives*. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators.

compressors and generators

- ***The belt drives are usually classified according to the cross section:***

a) Flat belt.

The flat belt, as shown in Fig. 6.1 (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 meters apart.

b) V-belt. The V-belt, as shown in Fig. 6.1 (*b*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

***c)Circular belt or rope*.** The circular belt or rope, as shown in Fig. 6.1 (*c*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 meters apart.

a) Flat belt *V-belt* *c) Circular belt or rope.*

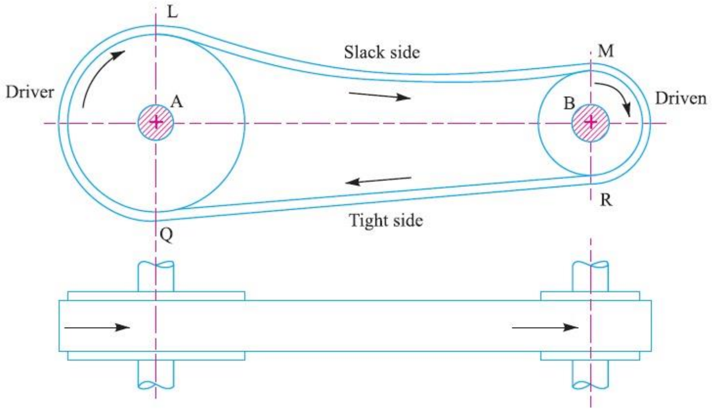
Fig 6.1

If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

5.3 Flat Belt Drives

a) Open belt drive.

The open belt drive, as shown in Figure below,

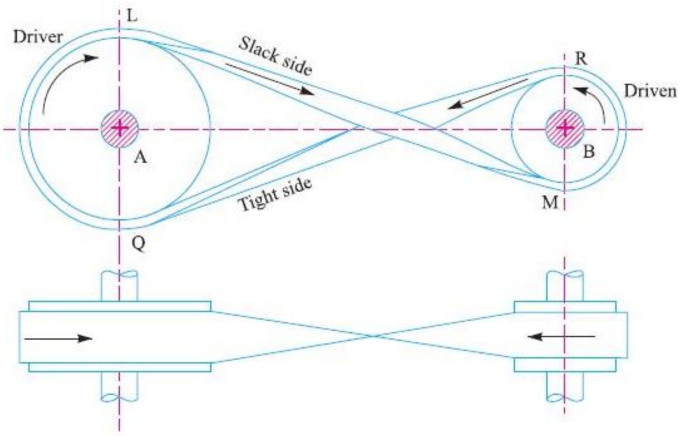


Open belt drive

is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.* lower side *RQ*) and delivers it to the other side (*i.e.* upper side *LM*). Thus, the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side***, as shown in Fig. 11.3.

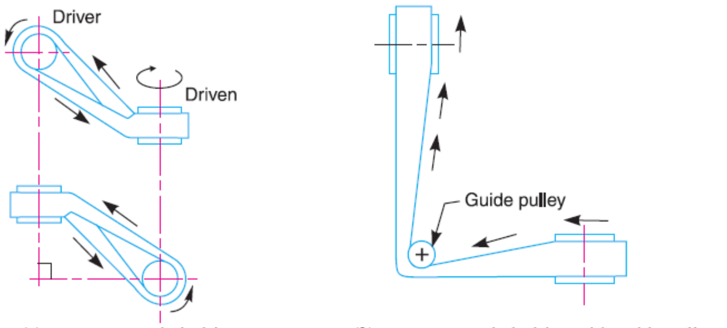
b) Crossed or twist belt drive.

The crossed or twist belt drive, as shown in Figure below, is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (*i.e.* *RQ*) and delivers it to the other side (*i.e.* *LM*). Thus, the tension in the belt *RQ* will be more than that in the belt *LM*. The belt *RQ* (because of more tension) is known as ***tight side,*** whereas the belt *LM* (because of less tension) is known as ***slack side***, as shown in Figure below.



c) Quarter turn belt drive.

 The quarter turn belt drive also known as right angle belt drive, as shown in Figre (a), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 b, where b is the width of belt. In case the pulleys cannot be arranged, as shown in Fig. 11.5 (a), or when the reversible motion is desired, then a **quarter turn belt drive with guide pulley**, as shown in Figure (b), may be used. Crossed or twist belt drive.

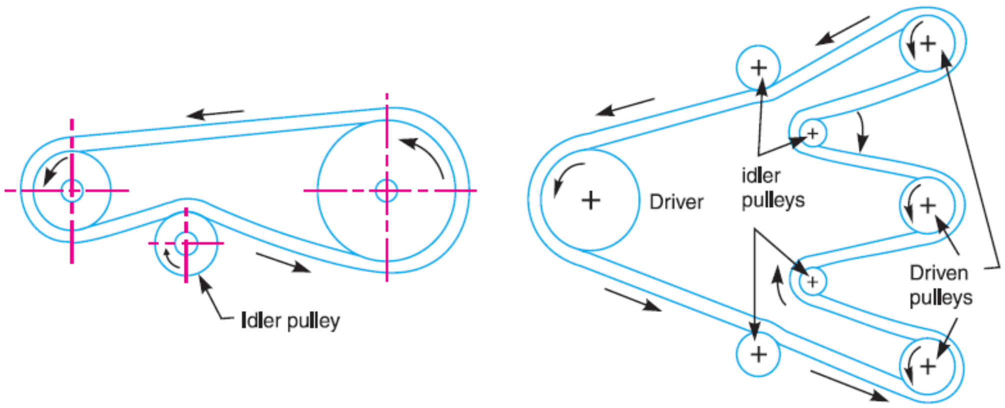


(*a*) Quarter turn belt drive. (*b*) Quarter turn belt drive with guide pulley.

d)Belt drive with idler pulleys.

A belt drive with an idler pulley, as shown in Figure (*a*), is used with shafts arranged parallel and when an open belt drive cannot be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension cannot be obtained by other means.

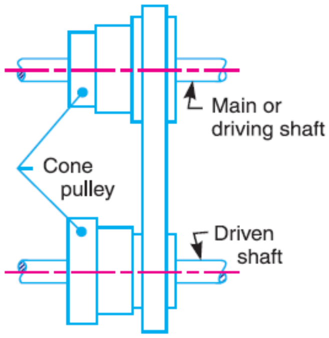
When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. 11.6 (*b*), may be employed.



(*a*) Belt drive with single idler pulley. (*b*) Belt drive with many idler pulleys.

e) Compound belt drive.

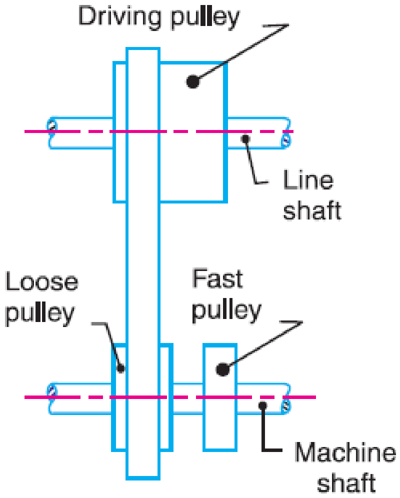
A compound belt drive, as shown in Figure below, is used when power is transmitted from one shaft to another through a number of pulleys.



Stepped or cone pulley drive.

***f) Fast and loose pulley drive*.**

A fast and loose pulley drive, as shown in Figure below, is used when the driven or machine shaft is to be started or stopped whenever desired without interfering with the driving shaft. A pulley which is keyed to the machine shaft is called ***fast pulley*** and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.



Fast and loose pulley drive.

5.4. Velocity Ratio of Belt Drive.

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let: $\boldsymbol{d_1}$ = Diameter of the driver, $\boldsymbol{d_2}$ = Diameter of the follower, $\boldsymbol{N_1}$ = Speed of the driver in r.p.m., and $\boldsymbol{N_2}$ = Speed of the follower in r.p.m.

∴ Length of the belt that passes over the driver, in one minute:

$$v_1 = \pi \cdot N_1 \cdot d_1$$

Similarly, length of the belt that passes over the follower, in one minute:

$$v_2 = \pi \cdot N_2 \cdot d_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore:

$$\pi \cdot N_1 \cdot d_1 = \mu \cdot N_2 \cdot d_2$$

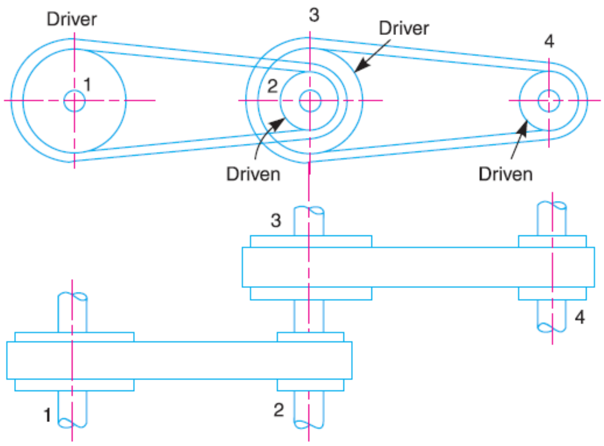
$$\therefore \text{Velocity ratio} \frac{N_1}{N_2} = \frac{d_2}{d_1}$$

When the thickness of the belt (***t***) is considered, then velocity ratio:

$$\frac{N_1}{N_2} = \frac{d_2 + t}{d_1 + t}$$

5.5. Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys as shown in Figure below. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.



Let: d_1 = Diameter of the pulley 1, n_1 = Speed of the pulley 1 in r.p.m., d_2, d_3, d_4, and n_2, n_3, n_4= Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2:

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \qquad (1)$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \qquad (2)$$

Multiplying equations **(1)** and **(2),**

$$\frac{N_2}{N_1} \cdot \frac{N_4}{N_3} = \frac{d_1}{d_2} \cdot \frac{d_3}{d_4}; (\ldots (\because \ N_2 \ = \ N_3, \text{being keyed to the same shaft})$$

$$\frac{N_4}{N_1} = \frac{d_1.d_3}{d_2 \cdot d_4}$$

A little consideration will show, that if there are six pulleys, then

$$\frac{N_6}{N_1} = \frac{d_1.d_3.d_5}{d_2 \cdot d_4 \cdot d_6}$$

Or

$$\frac{Speed\ of\ last\ driven}{Speed\ of\ first\ driver} = \frac{product\ of\ diameter\ of\ drivers}{roduct\ of\ diameter\ of\ drivens}$$

5.6. Slip of Belt

In the previous articles, we have discussed the motion of belts and shafts assuming a firm frictional grip between the belts and the shafts. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let: s_1 % = Slip between the driver and the belt, and s_2 % = Slip between the belt and the follower.

∴ Velocity of the belt passing over the driver per second:

$$v = \frac{\pi \cdot d_1 \cdot N_1}{60} - \frac{\pi \cdot d_1 \cdot N_1}{60} \cdot \frac{s_1}{100} = \frac{\pi \cdot d_1 \cdot N_1}{60} \cdot \left(1 - \frac{s_1}{100}\right) \tag{3}$$

and velocity of the belt passing over the follower per second:

$$\frac{\pi \cdot d_2 \cdot N_2}{60} = v - v \cdot \frac{s_2}{100} = v \cdot \left(1 - \frac{s_2}{100}\right)$$

Substituting the value of ***v*** from equation (3):

$$\frac{\pi \cdot d_2 \cdot N_2}{60} = \frac{\pi \cdot d_1 \cdot N_1}{60} \cdot \left(1 - \frac{s_1}{100}\right) \cdot \left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \cdot \left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right); \left(\text{Neglecting } \frac{s_1 \cdot s_2}{100 \cdot 100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \cdot \left(1 - \frac{s_1 + s_2}{100}\right) = \frac{d_1}{d_2} \cdot \left(1 - \frac{s}{100}\right); where\ s = s_1 + s_2\ (s\ is\ total\ persentage\ of\ slip$$

If thickness of the belt (t) is considered, then:

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t} \cdot \left(1 - \frac{s}{100}\right)$$

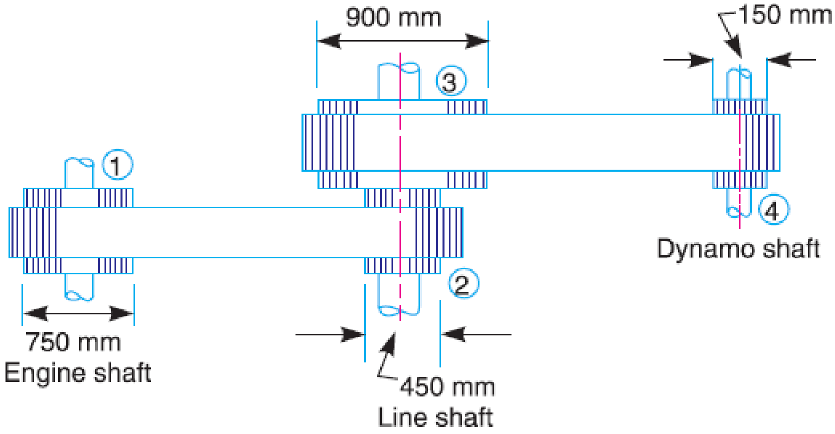
Example1

An engine, running at 150 r.p.m., drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when **1.** there is no slip, and **2.** there is a slip of 2% at each drive.

Solution.

Given: N_1 = 150 r.p.m.; d_1 = 750 mm; d_2 = 450 mm; d_3 = 900 mm; $d4$ = 150 mm

The arrangement of belt drive is shown in Figure below



Let n_4 = Speed of the dynamo shaft.

1. *When there is no slip*

We know that:

$$\frac{N_4}{N_1} = \frac{d_1.d_3}{d_2 \cdot d_4} \xrightarrow{yields} \frac{N_4}{150} = \frac{750 \cdot 900}{450 \cdot 150} = 10 \xrightarrow{yields} N_4 = 150 \cdot 10 = 1500\, rpm$$

2.When there is a slip of 2% at each drive

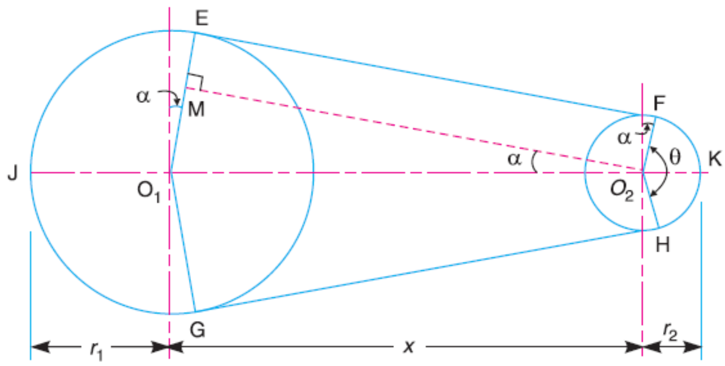
$$\frac{N_4}{N_1} = \frac{d_1.d_3}{d_2 \cdot d_4} \cdot \left(1 - \frac{s_1}{100}\right) \cdot \left(1 - \frac{s_2}{100}\right) \xrightarrow{yields} \frac{n_4}{150}$$

$$= \frac{750 \cdot 900}{450 \cdot 150} \cdot \left(1 - \frac{2}{100}\right) \cdot \left(1 - \frac{2}{100}\right)$$

$$N_4 = 150 \cdot 9.6 = 1440\, rpm$$

5.7. Determination of Angle of Contact (θ)

When the two pulleys of different diameters are connected by means of an open belt as shown in Figure below then the angle of contact or lap (θ) at the smaller pulley (driving pulley) must be taken into consideration.



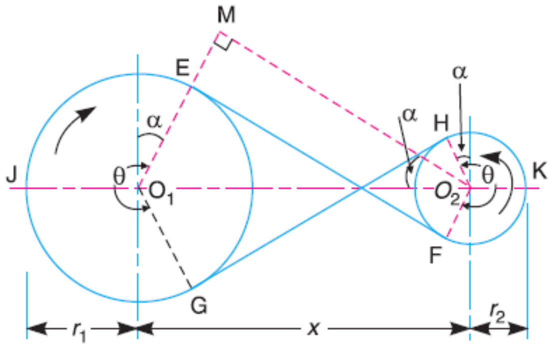
Open belt drive.

Let: r_1 = Radius of larger pulley, r_2 = Radius of smaller pulley, and x = Distance between centers of two pulleys (*i.e.* O_1 O_2).

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - ME}{O_1O_2} = \frac{r_1 - r_2}{x}$$

$$\therefore Angle\ open\ belt\ drive\ of\ contact\ or\ lap\ \theta = (180 - 2\cdot\alpha)\cdot\frac{\pi}{180} rad$$

A little consideration will show that when the two pulleys are connected by means of ***a crossed belt*** as shown in Figure then the angle of contact or lap (θ) on both the pulleys is same. From Figure:



Crossed belt drive.

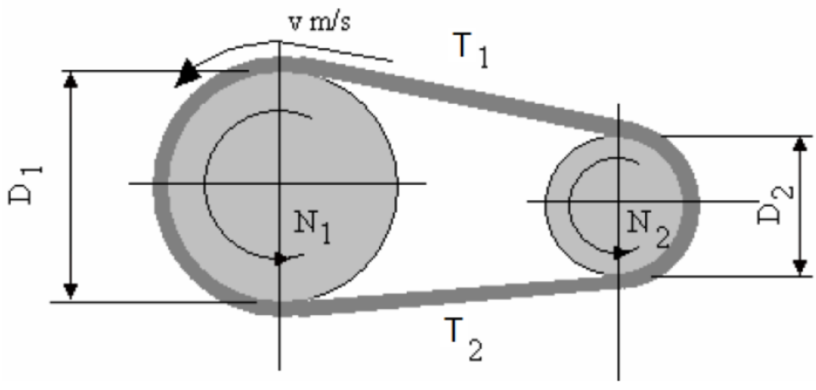
$$\sin\alpha = \frac{r_1 + r_2}{x}$$

$$\therefore Angle\ cross\ belt\ drive\ of\ contact\ or\ lap\ \theta = (180 + 2\cdot\alpha)\cdot\frac{\pi}{180} rad$$

5.8. Belt tension:

The tension in the belt will be the same along its whole length, and equal to T Or

$$T = T_1 + T_2 = constat$$



And the initial tension is:

$$T_0 = \frac{T_1 + T_2}{2}$$

5.9. Power transmitted

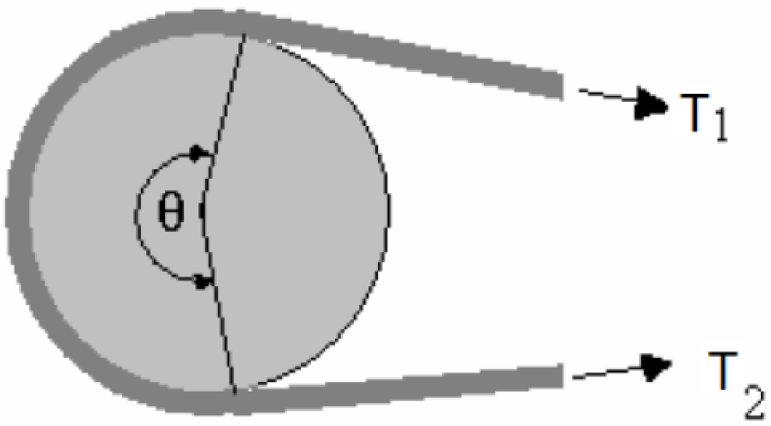
$$P_W = v\cdot(T_1 - T_2); for\ v = \pi\cdot N\cdot D \xrightarrow{yields} P_W = \pi\cdot N\cdot D\cdot(T_1 - T_2)$$

Where T_1 is tension for driving pulley, T_2 is tension for driven pulley

5.10 Relation of Driving Tension

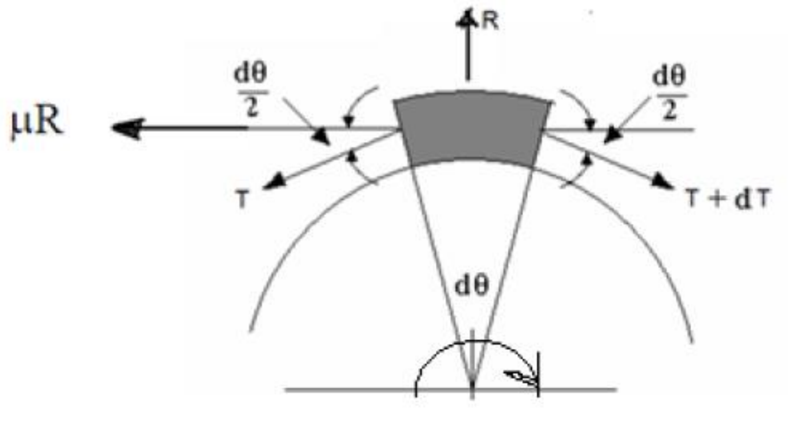
5.10.1. For flat Belt.

Consider a pulley wheel with a belt passing around it as shown. In order for the belt to produce a torque on the wheel (whether or not it is rotating), there must be tension in both ends. If this was not so, the belt would not be pressed against the wheel and it would slip on the wheel. The belt depends upon friction between it and the wheel in order to grip and produce torque.



For a belt to produce torque on the wheel, the force in one end must be greater than the force in the other end. otherwise, the net torque is zero.

Let $\mathbf{T_1}$ be the larger force and $\mathbf{T_2}$ the smaller force. $\boldsymbol{\theta}$ is the angle of lap. Now consider an elementary length of the belt on the wheel. The force on one end is **T** and on the other end is slightly larger and is **T + dT**. The angle made by the small length is **dθ**.



R is the reaction between the belt and the pulley.

The forces in the Radial direction (vertically) are:

$$T \cdot sin\frac{d\theta}{2} + (T + dT) \cdot \ sin\frac{d\theta}{2} = R; Since \frac{d\theta}{2} is\ small\ so\ sin\frac{d\theta}{2} \ = \ \frac{d\theta}{2}$$

Hence

$$T \cdot \frac{d\theta}{2} + T\ \frac{d\theta}{2} + dT \cdot \frac{d\theta}{2} = R; \text{since } dT \cdot \frac{d\theta}{2} \text{ is very small so it can be neglecting}$$

So

$$T \cdot d\theta = R \qquad (1)$$

The forces in the Tangential direction (Horizontally) are:

$$T + dT \cdot \; cos\frac{\theta}{2} - \; T \cdot cos\frac{\theta}{2} = \; \mu \cdot R \; ,$$

$$since \quad \sin\frac{\theta}{2} \text{ is very small } so \;\; cos\frac{\theta}{2} = 1$$

Then:

$$dT = \; \mu \cdot R \qquad (2)$$

Subst. eqn. (1) in eqn. (2) gives:

$$dT = \mu \cdot T \cdot d\theta \; or \; \frac{dT}{T} = \; \mu \cdot d\theta$$

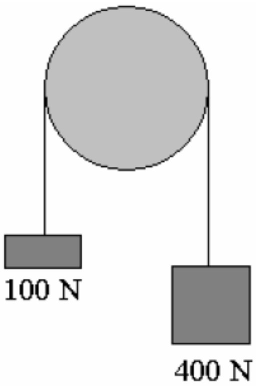
$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu \cdot d\theta \xrightarrow{yields} ln\frac{T_1}{T_2} = \mu \cdot \theta$$

$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} \qquad (3)$$

Where T_1 is tension for driving pulley, T_2 is tension for driven pulley, μ is coefficient friction between contact surface belt and pulley, θ is the angle of lap.

Example2

A rope is hung over a stationary drum as shown with weights on each. The rope is just on the point of slipping. What is coefficient of frication?



Solution

Angle of lap $\theta = 180^0 = \pi$; T_1=400 N; T_1=100 N.

$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} \xrightarrow{yields} \frac{400}{100} = e^{\mu \cdot \pi} \xrightarrow{yields} ln4 = \pi \cdot \mu \xrightarrow{yields} \mu = \frac{ln4}{\pi}$$

μ=0.441

Example3

Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and the pulley is 0.25, angle of lap160° and maximum tension in the belt is 2500 N.

Solution. Given: d = 600 mm = 0.6 m; N = 200 r.p.m.; μ = 0.25;

θ = 160° = 160 × π / 180= 2.793 rad; T_1 = 2500 N

We know that velocity of the belt,

$$v = \frac{\pi \cdot d \cdot N}{60} = \frac{\pi \cdot 0.6 \cdot 200}{60} = 6.284\, m/s$$

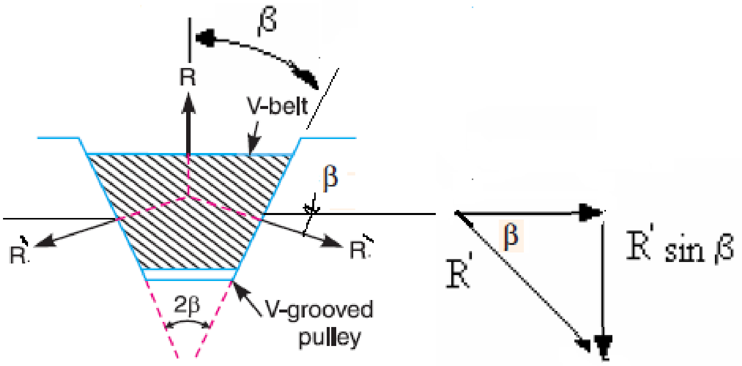
T_2 = Tension in the slack side of the belt.

$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} \xrightarrow{yields} \frac{2500}{T_2} = e^{0.25 \cdot 2.93} \xrightarrow{yields} T_2 = \frac{T_1}{e^{0.25 \cdot 2.93}} = \frac{2500}{e^{0.25 \cdot 2.93}} = 1201\ N$$

$$P_W = v \cdot (T_1 - T_2) = 6.284 \cdot (2500 - 1201) = 8162.9\ W$$

5.10.2. For VEE-Belts

Consider a VEE-Belt with an included angle of 2β. V-belt is grip on the side not on the bottom as shown below:



$$\acute{R} \cdot \sin\beta = \frac{1}{2} \cdot R \xrightarrow{yields} \acute{R} = \frac{R}{2 \cdot sin\,\beta}$$

In the flat belt we have:

$$dT = \mu \cdot R \xrightarrow{yields} for\ vee\ belt\ dT = \mu \cdot \acute{R}$$

$$\text{For to side } dT = 2 \cdot \mu \cdot \acute{R} \xrightarrow{yields} dT = \frac{\mu \cdot R}{sin\,\beta}$$

$$\text{Let} \acute{\mu} = \frac{\mu}{sin\,\beta} \xrightarrow{yields} dT = \acute{\mu} \cdot R$$

$$\text{For } T \cdot d\theta = R$$

$$\text{So } dT = \acute{\mu} \cdot T \cdot d\theta \xrightarrow{yields} \int_{T_2}^{T_1} \frac{dT}{T} = \acute{\mu} \int_0^{\theta} d\theta$$

$$ln \frac{T_1}{T_2} = \acute{\mu} \cdot \theta$$

$$\frac{T_1}{T_2} = e^{\acute{\mu}\theta} = e^{\frac{\mu \cdot \theta}{sin\,\beta}}$$

Where T_1 is tension for driving pulley, T_2 is tension for driven pulley, μ is coefficient friction between contact surface belt and pulley, θ is the angle of lap.

5.11. Maximum power transmitted by pulleys

The force (tension) in a pulley belt increases with torque and power. The maximum power that a pulley system can transmit is ultimately limited by the strength of the belt material. If this is a problem then more than one belt should be used to share the load. If the belt does not break then the possibility of the belt slipping exists and this depends upon the angle of lap and the coefficient of friction. If the coefficient of friction is the same on both wheels, then slippage will occur first on the smaller wheel. The power at which the belt slips is not the absolute maximum power that can be transmitted as more power can be transmitted with slippage occurring by using higher wheel speeds.

The friction between the belt and the wheel is further affected by centrifugal force which tends to lift the belt off the wheel. This increases the likelihood of slippage.

The friction between the belt and wheel may be increased by the shape of the belt. A vee section or round section belt in a vee groove will grip better than a flat belt and is less likely to slip.

5.11.1. Maximum power with no belt slip.

Equation (3) is used when the belt starts to slip. So, the power transmitted:

$$P_W = \pi \cdot N \cdot D \cdot (T_1 - T_2)$$

When the belt started to slip, the force ratio is:

$$T_2 = T_1 \cdot e^{-\mu \cdot \theta}$$

then the power is:

$$P_W = \pi \cdot N \cdot D \cdot T_1 \cdot \left(1 - e^{-\mu \cdot \theta}\right) \; or \; P_W = v \cdot T_1 \cdot \left(1 - e^{-\mu \cdot \theta}\right)$$

Where N is number of rotating a pulley [rpm]; D is diameter pullet [m]; μ is coefficient frication contact surface belt and pulley [-]; θ is angle of contact of the belt on the driving pulley; T_1 is a tension fore in direction the angle of lap [N]; T_2 is tension force in opposite direction of the angle of lap [N].

The number of belts or ropes required (***n***) can be obtained by:

$$n = \frac{Total\ Power\ Tansmitted}{Power\ transmitted\ per\ belt\ or\ rope}$$

Example 4

The tension in a pulley belt is 110N when stationary. Calculate the tension in each side and the power transmitted when the belt is on the point of slipping on the smaller wheel. The wheel is 240 mm diameter and the coefficient of friction is 0.32. The angle of lap is 165^0. The wheel speed is 1500 rev/min.

Solution

Belt velocity: $v = \pi \cdot N \cdot D = \pi \cdot \frac{1500}{60} \cdot 0.24 = 18.85\ m/s$

Lap Angle: $\theta = \frac{165}{180} \cdot \mu = 2.87\ rad$

Initial tensions: $T_1 = T_2 = 110\ N\ when\ stationary\ so\ T_1 + T_2 = 220N$

$\xrightarrow{yields} T_2 = 220N - T_1$

Tension in belts:

$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} = e^{0.32 \cdot 2.87} = 2.513$$

$$T_1 = 2.513 \cdot T_2 = 2.513 \cdot (220N - T_1) = 552.9 - 2.513 \cdot T_1$$

$$T_1 + 2.513 \cdot T_1 = 552.9 \xrightarrow{yields} T_1 = 157.4N; T_2 = \frac{157.4}{2.513} = 62.6\ N$$

Power: $P = v \cdot (T_1 - T_2) = 18.85 \cdot (\ 157.4 - 62.6\ ) = 1786.W$

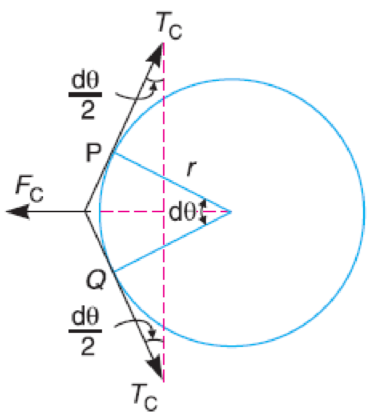
Check: $P_W = v \cdot T_1 \cdot \left(1 - e^{-\mu \cdot \theta}\right) = 18.85 \cdot 157.4 \cdot (1 - 0.398) = 1786.13\ W$

5.12. The effect of centrifugal tension(force):

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension***. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a belt of mass m per unit length, and an element of this belt can be shown in the figure bellow:

Consider a small portion *PQ* of the belt subtending an angle *d*θ the centre of the pulley as shown in Figure below:



Let:

m = Mass of the belt per unit length in kg, ***v*** = Linear velocity of the belt in m/s,
r = Radius of the pulley over which the belt runs in meters, and
$\boldsymbol{T}_{\mathrm{C}}$ = Centrifugal tension acting tangentially at P and Q in newtons.
We know that length of the belt PQ.:

$$PQ = r \cdot d\theta$$

∴ Centrifugal force acting on the belt:

$$m_{PQ} = m \cdot PQ = m \cdot r \cdot d\theta$$

∴ Centrifugal force acting on the belt PQ:

$$F_c = m_{PQ} \cdot \omega^2 \cdot r = m \cdot r \cdot d\theta \cdot \frac{v^2}{r^2} \cdot r = m \cdot v^2 \cdot d\theta$$

The centrifugal tension T_{C} acting tangentially at P and Q keeps the belt in equilibrium. Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally and equating the same, we have:

$$T_c \cdot \sin\frac{d\theta}{2} + T_c \cdot \sin\frac{d\theta}{2} = F_c = m \cdot v^2 \cdot d\theta$$

Since the angle ***d*θ** is very small, therefore, putting: $sin\frac{d\theta}{2} = \frac{d\theta}{2}$ in the above expression

$$2 \cdot T_c \cdot \frac{d\theta}{2} = m \cdot v^2 \cdot d\theta \xrightarrow{yields} T_c = m \cdot v^2$$

Notes:

1. When the centrifugal tension is taken into account, then total tension in the tight side:

$$T_{t1} = T_1 + T_c$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_c$$

2. Power transmitted:

$$P_W = v \cdot (T_{t1} - T_{t2}) = v \cdot [(T_1 + T_c) - (T_2 + T_c)] = v \cdot (T_1 - T_2)$$

Thus, we see that centrifugal tension has no effect on the power transmitted.

3. The ratio of driving tensions may also be written as:

$$\frac{T_{t1} - T_c}{T_{t2} - T_c} = e^{\mu \cdot \theta} \text{ (for flat belts.)}$$

$$\frac{T_{t1} - T_c}{T_{t2} - T_c} = e^{\frac{\mu \cdot \theta}{\sin\beta}} \text{ (for VEE − Belts.)}$$

6.13 Condition for the Transmission of Maximum Power

We know that power transmitted by a belt:

$$P = v \cdot (T_1 - T_2) \qquad \text{(I)}$$

Where T_1 = Tension in the tight side of the belt in newtons, T_2 = Tension in the slack side of the belt in newtons, and v = Velocity of the belt in m/s. we have also seen that the ratio of driving tensions is:

$$\frac{T_1}{T_2} = e^{\mu\cdot\theta} \xrightarrow{yields} T_2 = \frac{T_1}{e^{\mu\cdot\theta}} \qquad \text{(II)}$$

Substituting the value of T_2 in equation (&)

$$P = v\cdot\left(T_1 - \frac{T_1}{e^{\mu\cdot\theta}}\right) = v\cdot T_1\cdot\left(1-\frac{1}{e^{\mu\cdot\theta}}\right) = v\cdot T_1\cdot C \qquad \text{(III)}$$

Where:

$$C =\cdot \left(1-\frac{1}{e^{\mu\cdot\theta}}\right)$$

We know that:

$$T_1 = T - T_c$$

Where:

T = Maximum tension to which the belt can be subjected in newtons, and T_C = Centrifugal tension in newtons. Substituting the value of T_1 in equation **(III),**

$$P = v\cdot C\cdot(T - T_c) = v\cdot C\cdot(T - m\cdot v^2), (substituting\ T_c = m\cdot v^2)$$

$$P = C\cdot(T\cdot v - m\cdot v^3)$$

For maximum power, differentiate the above expression with respect to v and equate to zero,

$$\frac{dP}{dv} = 0 \xrightarrow{yields} \frac{d}{dv}C\cdot(T\cdot v - m\cdot v^3) = 0 \xrightarrow{yields} T - 3\cdot m\cdot v^2 = 0$$

$$T = 3\cdot T_c = 3\cdot m\cdot v^2 \qquad \text{(IV)}$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

Notes: 1. We know that:

$$T_1 = T - T_c, and\ for\ maximum\ power T_c = \frac{T}{3}$$

$$\therefore T_1 = T - \frac{T}{3} = \frac{2\cdot T}{3}$$

2. From equation ***(IV),*** the velocity of the belt for the maximum power,

$$v = \sqrt{\frac{T}{3\cdot m}}$$

Example5

A shaft rotating at 200 r.p.m. drives another shaft at 300 r.p.m. and transmits 6 kW through a belt. The belt is 100 mm wide and 10 mm thick. The distance between the shafts is 4m. The smaller pulley is 0.5 m in diameter. Calculate the *stress in the belt, if it is* **1.** *an open belt drive, and* **2.** *across belt drive. Take μ = 0.3.*

Solution. Given: N_1 = 200 r.p.m.; N_2 = 300 r.p.m.; P = 6 kW = 6 × 10^3 W; b = 100 mm; t = 10 mm; x = 4 m; d_2 = 0.5 m; μ = 0.3.

Let:

σ = Stress in the belt.

1. *Stress in the belt for an open belt drive*:

First of all, let us find out the diameter of larger pulley (d_1). We know that:

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \xrightarrow{yields} d_1 = d_2 \cdot \frac{N_2}{N_1} = 0.5 \cdot \frac{300}{200} = 0.75m$$

and velocity of the belt,

$$v = \frac{\pi \cdot d_2 \cdot N_2}{60} = \frac{\pi \cdot 0.5 \cdot 300}{60} = 7.85\, m/s$$

Now let us find the angle of contact on the smaller pulley. We know that, for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2 \cdot x} = \frac{0.75 - 0.5}{2 \cdot 4} = 0.03125 \xrightarrow{yields} \alpha = 1.8°$$

$$\therefore Angle\ of\ contact\ or\ lap\ \theta = (180 - 2 \cdot \alpha) \cdot \frac{\pi}{180} rad$$

$$= (180 - 2 \cdot 1.8) \cdot \frac{\pi}{180} rad = 3.08 rad$$

Let: T_1 = Tension in the tight side of the belt, and T_2 = Tension in the slack side of the belt.

We know that:

$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} = e^{0.3 \cdot 3.08} = 2.52 \qquad (1)$$

We also know that power transmitted (P):

$$P = v \cdot (T_1 - T_2) \xrightarrow{yields} 6 \times 10^3 = 7.85 \cdot (T_1 - T_2)$$

$$(T_1 - T_2) = \frac{6 \times 10^3}{7.85} = 764.33 \qquad (2)$$

From equations (*1*) and (*2*),

$$T_1 = 1267\, N, and\ T_2 = 503\, N$$

We know that maximum tension in the belt (T_1):

$$\sigma = \frac{T_1}{A_{belt}} = \frac{1267}{b \cdot t} = \frac{1267}{100 \cdot 10} = 1.267\ N/mm^2 = 1.267\ MPa$$

2. Stress in the belt for a cross belt drive

We know that for a cross belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{d_1 + d_2}{2 \cdot x} = \frac{0.75 + 0.5}{2 \cdot 4} = 0.1562 \xrightarrow{yields} \alpha = 9°$$

$$\therefore Angle\ of\ contact\ or\ lap\ \theta = (180 + 2 \cdot \alpha) \cdot \frac{\pi}{180} rad$$
$$= (180 + 2 \cdot 9) \cdot \frac{\pi}{180} rad = 3.4 rad$$
We know that:
$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} = e^{0.3 \cdot 3.456} = 2.82 \quad (3)$$
From equations (*2*) and (*3*):
$$T_1 = 1184\ N, and\ T_2 = 420\ N$$
$$\sigma = \frac{T_1}{A_{belt}} = \frac{1184}{b \cdot t} = \frac{1184}{100 \cdot 10} = 1.184\ N/mm^2 = 1.184\ MPa$$
Example 6

A leather belt is required to transmit 7.5 kW from a pulley 1.2 m in diameter, running at 250 r.p.m. The angle embraced is 165° and the coefficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa, density of leather 1 Mg/m3 and thickness of belt 10 mm, determine the width of the belt taking centrifugal tension into account.

Solution.

Given: P = 7.5 kW = 7500 W; d = 1.2 m; N = 250 r.p.m.; θ = 165° = 165 × π / 180 = 2.88 rad ; μ = 0.3 ; σ = 1.5 MPa = 1.5×10^6 N/m^2 ; ρ = 1 Mg/m^3 = 1×10^6 g/m^3 = 1000 kg/m^3; t = 10 mm = 0.01 m

Let:

b = Width of belt in meters, T_1 = Tension in the tight side of the belt in N, and T_2 = Tension in the slack side of the belt in N.

We know that velocity of the belt:
$$v = \frac{\pi \cdot d\ .N}{60} = \frac{\pi \cdot 1.2\ .250}{60} = 15.71\ m/s$$
and power transmitted (P),
$$P = v \cdot (T_1 - T_2) \xrightarrow{yields} 7500 = 15.71 \cdot (T_1 - T_2)$$
$$\therefore (T_1 - T_2) = \frac{7500}{15.71} = 477.4 \quad (1)$$
We know that
$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} = e^{0.3 \cdot 2.88} = 2.375 \quad (2)$$
From equations **(*1*)** and **(2),** T_1 = 824.6 N, and T_2 = 347.2 N

We know that mass of the belt per meter length,
$$m = Area \cdot length \cdot density = b \cdot t \cdot l \cdot \rho = b \cdot 0.01 \cdot 1 \cdot 1000$$
$$= 10\ b\ \text{kg}$$
∵Centrifugal tension:

$$T_c = m \cdot v^2 = 10 \cdot \ b \cdot 15.71^2 = 157.1 \cdot b = 2468 \cdot b$$

and maximum tension in the belt

$$T \ = \ \sigma.b.t \ = \ 1.5 \ \times \ 10^6 \ \cdot \ b \ \cdot \ 0.01 \ = \ 15\,000 \ \cdot b\,N$$

We know that:

$$T \ = \ T_{\ 1} + \ T_C \ or\ 15000 \ \cdot b$$
$$= \ 824.6 \ + \ 2468\,b$$
$$\xrightarrow{yields} 15\,000 \ \cdot b - \ 2468 \cdot \ b \ = \ 824.6 \xrightarrow{yields} \ 12\,532 \ \cdot b \ = \ 824.6$$

$$b \ = \ \frac{824.6}{12\,532} = 0.0658\,m \ = \ 65.8\,mm$$

Example 7

A compressor, requiring 90 kW is to run at about 500 r.p.m. The drive is by V-belts from an electric motor running at 1500 r.p.m. The diameter of the pulley on the compressor shaft must not be greater than 1 meter while the center distance between the pulleys is limited to 1.75 meter.

Determine the number of V-belts required to transmit the power if each belt has a cross-sectional area of 375 mm^2, density 1000 kg/m^3 and an allowable tensile stress of 2.5 MPa. The groove angle of the pulley is 35°. The coefficient of friction between the belt and the pulley is 0.25. Calculate also the length required of each belt.

Solution. Given : P = 90 kW ; n_2 = 250 r.p.m. ; N_1 = 750 r.p.m. ; d_2 = 1 m ; x = 1.75 m ; v = 1600 m/min = 26.67 m/s ; a = 375 mm^2 = 375 × 10^{-6} m^2 ; ρ = 1000 kg/m^3 ; σ = 2.5 MPa = 2.5 × 10^6 N/m^2 ; 2 β = 35° or β = 17.5° ; μ = 0.25

First of all, let us find the diameter of pulley on the motor shaft (d_1). We know that:

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \xrightarrow{yields} d_1 = \frac{N_2 \cdot d_2}{N_1} = \frac{500 \cdot 1}{1500} = 0.33m$$

$$v = \frac{\pi \cdot d_2 \,.\, N}{60} = \frac{\pi \cdot 1 \,\ .500}{60} = 26.17\, m/s$$

We know that the mass of the belt per metre length:

$$m = Area \cdot length \cdot density = 375 \times 10^{-6} \cdot 1 \cdot 1000 = 0.375\, kg$$

∴ Centrifugal tension:

$$T_c = m \cdot v^2 = 0.375 \cdot 26.67\ ^2 = 257\ N$$

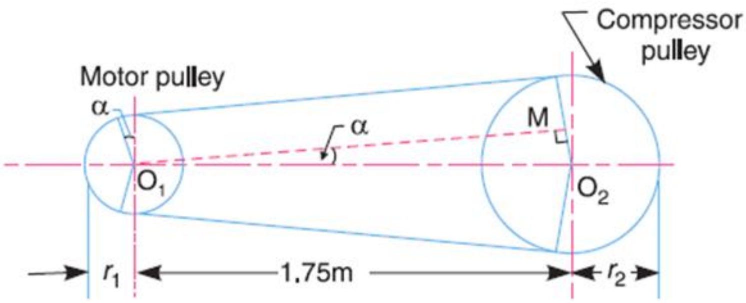
and maximum tension in the belt:

$$T = \sigma \cdot A = 2.5 \cdot 10^6 = 937.5\, N$$

∴ Tension in the tight side of the belt:

$$T_1 = T - T_C = 937.5 - 257 = 680.5\, N$$

For an open belt drive, as shown in Figure:



$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2 \cdot x} = \frac{1 - 0.33}{2 \cdot 1.75} = 0.1914 \xrightarrow{yields} \alpha = 11°$$

and angle of lap on smaller pulley (i.e. pulley on motor shaft):

$$\therefore \text{Angle of contact or lap } \theta = (180 - 2 \cdot \alpha) \cdot= (180 - 2 \cdot 11) \cdot= 158°$$

$$\theta = 158 \cdot \frac{\pi}{180} = 2.76 rad$$

We know that:

$$\frac{T_1}{T_2} = e^{\frac{\mu \cdot \theta}{\sin \beta}} = e^{\frac{0.25 \cdot 2.7}{\sin 17.5}} = 9.43$$

$$\frac{680.5}{T_2} = 9.43 \xrightarrow{yields} T_2 = \frac{680.5}{9.954} = 71 N$$

Number of V-belts

We know that power transmitted per belt:

$$P = v \cdot (T_1 - T_2) = 26.67 \cdot (680.5 - 71) = 16255\, W$$

$$\text{Number of } V - \text{belts} = \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{90000}{16255} = 5.5 \approx 6$$

Length of each belt

We know that length of belt for an open belt drive:

$$L = \pi \cdot (r_1 + r_2) + 2 \cdot x + \frac{(r_1 - r_2)^2}{x} = \frac{\pi}{2} \cdot (d_1 + d_2) + 2 \cdot x + \frac{(d_1 - d_2)^2}{4 \cdot x}$$

$$L = \frac{\pi}{2} \cdot (1 + 0.33) + 2 \cdot 1.75 + \frac{(1 - 0.33)^2}{4 \cdot 1.75} =$$

$$L = 2.1 + 3.5 + 0.064 = 5.664\, m$$

Example 8

Determine the width of a 9.75 mm thick leather belt required to transmit 15 kW from a motor running at 900 r.p.m. The diameter of the driving pulley of the motor is 300 mm. The driven pulley runs at 300 r.p.m. and the distance between the center of two pulleys is 3 meters. The density of the leather is 1000 kg/m^3. The maximum allowable stress in the leather is 2.5 MPa. The coefficient of friction between the leather and pulley is 0.3. Assume open belt drive and neglect the sag and slip of the belt.

Solution. Given : t = 9.75 mm = 9.75 × 10^{-3} m ; P = 15 kW = 15 × 103 W ; n_1 = 900 r.p.m. ; d_1 = 300 mm = 0.3 m ; n^2 = 300 r.p.m. ; x = 3m ; ρ = 1000 kg/m^3 ; σ = 2.5 MPa = 2.5 × 10^6 N/m^2 ; μ = 0.3

First of all, let us find out the diameter of the driven pulley (d_2) and velocity. We know that:

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \xrightarrow{yields} d_2 = \frac{N_1 \cdot d_1}{N_2} = \frac{900 \cdot 0.3}{300} = 0.9\, m$$

$$v = \frac{\pi \cdot d_1 \cdot N_1}{60} = \frac{\pi \cdot 0.3 \cdot 900}{60} = 14.14\, m/s$$

For an open belt drive:

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2 \cdot x} = \frac{0.9 - 0.3}{2 \cdot 3} = 0.1 \xrightarrow{yields} \alpha = 5.74°$$

$$\therefore Angle\ of\ contact\ or\ lap\ \theta = (180 - 2 \cdot \alpha) \cdot= (180 - 2 \cdot 5.74) \cdot= 168.52°$$

$$\theta = 168.52 \cdot \frac{\pi}{180} = 2.94\, rad$$

$$\frac{T_1}{T_2} = e^{\mu \cdot \theta} = e^{0.3 \cdot 2.94} = 2.42 \qquad (1)$$

We also know that power transmitted (P):

$$P = v \cdot (T_1 - T_2) \xrightarrow{yields} 15000 = 14.14 \cdot (T_1 - T_2)$$

$$T_1 - T_2 = \frac{15000}{14.14} = 1060N \qquad (2)$$

From equations **(*1*)** and **(*2*):**

$$T_1 = 1806\, N, T_2 = 746.2N$$

b = Width of the belt in metres

We know that mass of the belt per metre length,

$$mass = Area \cdot length \cdot density = b\ \cdot\ 9.75 \cdot 10^{-3}\ \times\ 1\ \times\ 1000 = 9.75\, bkg$$

Centrifugal tension

$$T_c = m \cdot v^2 = 9.75\, b \cdot 14.14\ ^2 = 1950\, bN$$

Maximum tension in the belt:

$$T = \sigma \cdot A = \sigma \cdot b \cdot t = 2.5\ \cdot\ 10^6\ \cdot\ b\ \cdot\ 9.75\ \cdot\ 10^{-3} = 24\,400\, b\, N$$

We know that:

$$T = T_1 + T_C \xrightarrow{yields} 24\,400\, b = 1060 + 1950\, b,$$
$$22\,450\, b \;=\; 1806$$
$$b = \frac{1806}{22\,450} = 0.080\, m$$

5.14 Chain Drives

We have seen in belt and rope drives that slipping may occur. In order to avoid slipping, steel chains are used. The chains are made up of rigid links which are hinged together in order to provide the necessary flexibility for warping around the driving and driven wheels. The wheels have projecting teeth and fit into the corresponding recesses, in the links of the chain as shown in Figure bellow. The wheels and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio. The toothed wheels are known as ***sprocket wheels*** or simply ***sprockets***. These wheels resemble to spur gears.

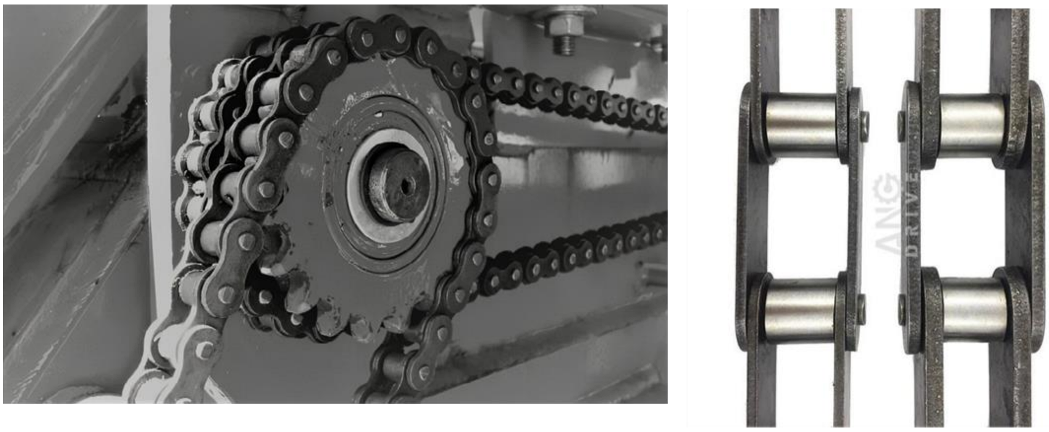


Figure of chain drive

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt
 or rope drive.
3. The chain drives may be used when the distance between the shafts is less.
4. The chain drive gives a high transmission efficiency (upto 98 per cent).
5. The chain drive gives less load on the shafts.
6. The chain drive has the ability of transmitting motion to several shafts by one chain only.

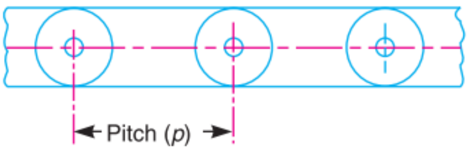
Disadvantages

1. The production cost of chains is relatively high.

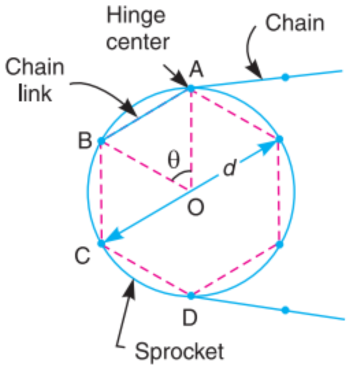
2. The chain drive needs accurate mounting and careful maintenance.
3. The chain drive has velocity fluctuations especially when unduly stretched.

5.15 Terms Used in Chain Drive

The following terms are frequently used in chain drive.
1. Pitch of the chain : It is the distance between the hinge center of a link and the corresponding hinge center of the adjacent link as shown in Figure It is usually denoted by ***p***
2. Pitch circle diameter of the chain sprocket.
It is the diameter of the circle on which the hinge centers of the chain lie, when the chain is wrapped round a sprocket as shown in Fig.. The points A, B, C, and ***D*** are the hinge centres of the chain and the circle drawn through these centres is called pitch circle and its diameter (***d***) is known as pitch circle diameter.



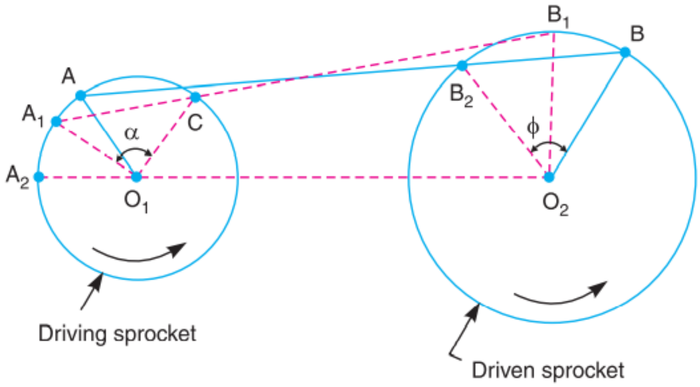
Pitch of the chain.



Pitch circle diameter of the chain sprocket.

5.16 Kinematic of Chain Drive

Fig. shows an arrangement of a chain drive in which the smaller or driving sprocket has 6 teeth and the larger or driven sprocket has 9 teeth. Though this is an impracticable case, but this is considered to bring out clearly the kinematic conditions of a chain drive. Let both the sprockets rotate anticlockwise and the angle subtended by the chain pitch at the centre of the driving and driven sprockets be α and φ respectively. The lines A B and A_1B_1 show the positions of chain having minimum and maximum inclination respectively with the line of centres O_1O_2 of the sprockets. The points A, B_2 and B are in one straight line and the points A_1,C and B_1 are in one straight line. It may be noted that the straight length of the chain between the two sprockets must be equal to exact number of pitches.



Kinematic of chain drive.

arrangement of a chain drive

Let

$$T1 \ = \ Number\ of\ teeth\ on\ the\ larger\ sprocket$$

$$T2 \ = \ Number\ of\ teeth\ on\ the\ smaller\ sprocket, and$$

$$p \ = \ Pitch\ of\ the\ chain$$

$$d = p \cdot cosec \frac{180}{T} \xrightarrow{yields} r = \frac{d}{2} = \frac{p}{2} \cdot cosec \frac{180}{T}$$

∴ For larger sprocket,

$$r_1 = \frac{p}{2} \cdot cosec \frac{180}{T_1}$$

and for smaller sprocket,

$$r_2 = \frac{p}{2} \cdot cosec \frac{180}{T_2}$$

Circular pitch..

$$p = \frac{\pi \cdot D}{T}$$

For larger sprocket

$$p = \frac{\pi \cdot D_1}{T_1} = \frac{2 \cdot \pi \cdot r_1}{T_1} \xrightarrow{yields} r_1 = \frac{p \cdot T_1}{2 \cdot \pi}$$

and for smaller sprocket,

$$p = \frac{\pi \cdot D_2}{T_2} = \frac{2 \cdot \pi \cdot r_2}{T_2} \xrightarrow{yields} r_2 = \frac{p \cdot T_2}{2 \cdot \pi}$$

5.16.1 Velocity Ratio of Chain Drive.

$$\frac{N_1}{N_2} = \frac{r_2}{r_1} = \frac{\frac{p \cdot T_2}{2 \cdot \pi}}{\frac{p \cdot T_1}{2 \cdot \pi}} = \frac{T_2}{T_1}$$

$$N_1 \cdot T_1 = N_2 \cdot T_2$$

5.16.2 Length of Chain

Since the term π ($r_1 + r_2$) is equal to half the sum of the circumferences of the pitch circles:

Let O1 chain circumference large sprocket and O2 chain circumference small sprocket.

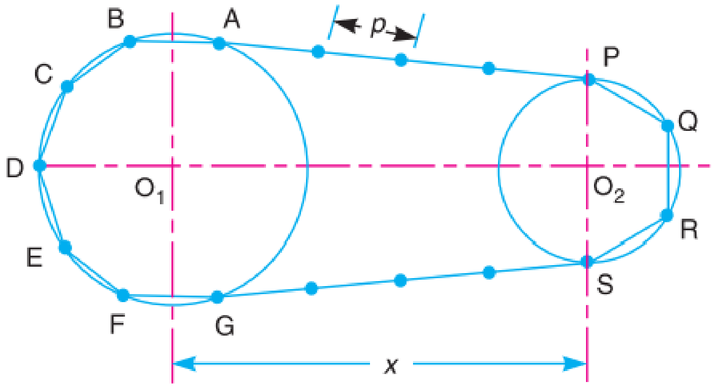
$$\frac{1}{2} \cdot (O_1 + O_2) = \frac{1}{2} \cdot (2 \cdot \pi \cdot r_1 + 2 \cdot \pi \cdot r_2) = \pi \cdot (r_1 + r_2) \cdot$$

therefore, the length of chain corresponding to:

$$\pi \cdot (r_1 + r_2) = \pi \cdot (\frac{p \cdot T_1}{2 \cdot \pi} + \frac{p \cdot T_2}{2 \cdot \pi}) = \frac{p \cdot (T_1 + T_2)}{2}$$

Length of Chain

$$L = \pi \cdot (r_1 + r_2) + 2 \cdot x + \frac{(r_1 - r_2)^2}{x}$$



Length of chain

Substituting the values of r_1 , r_2 and π ($r_1 + r_2$) in equation of length chain, the length of chain is given by:

$$L = \frac{p \cdot (T_1 + T_2)}{2} + 2 \cdot x + \frac{\left(\frac{p}{2} \cdot cosec \frac{180}{T_1} - \frac{p}{2} \cdot cosec \frac{180}{T_2}\right)^2}{x}$$

$$L = \frac{p \cdot (T_1 + T_2)}{2} + 2 \cdot x + \frac{p^2}{4 \cdot x} \cdot (cosec\frac{180}{T_1} - cosec\frac{180}{T_2})^2$$

Example 9

A chain drive is used for reduction of speed from 240 r.p.m. to 120 r.p.m. The number of teeth on the driving sprocket is 20. Find the number of teeth on the driven sprocket. If the pitch circle diameter of the driven sprocket is 600 mm and centre to centre distance between the two sprockets is 800 mm, determine the pitch and length of the chain.

Solution. Given : N_1 = 240 r.p.m ; N_2 = 120 r.p.m ; T_1 = 20 ; d_2 = 600 mm or r_2 = 300 mm = 0.3 m ; x = 800 mm = 0.8 m

Let T_2 = Number of teeth on the driven sprocket. We know that

$$N_1 \cdot T_1 = N_2 \cdot T_2 \xrightarrow{yields} T_2 = \frac{N_1 \cdot T_1}{N_2} = \frac{240 \cdot 20}{120} = 40\ tooth$$

Pitch of the chain

$$p = \frac{\pi \cdot D_2}{T_2} = \frac{2 \cdot \pi \cdot r_2}{T_2} = \frac{2 \cdot \pi \cdot 300}{40} = 47.1\ mm$$

Another solution

$$r_2 = \frac{p}{2} \cdot cosec\frac{180}{T_2} \xrightarrow{yields} 300 = \frac{p}{2} \cdot cosec\frac{180}{40}$$

$$p = \frac{600}{cosec4.5} = 47.1mm$$

Length of the chain

$$r_1 = \frac{p}{2} \cdot cosec\frac{180}{T_1} = \frac{47.1}{2} \cdot cosec\frac{180}{20} = 150.5\ mm$$

$$L = \frac{p \cdot (T_1 + T_2)}{2} + 2 \cdot x + \frac{p^2}{4 \cdot x} \cdot (cosec\frac{180}{T_1} - cosec\frac{180}{T_2})^2$$

$$= \frac{47.1 \cdot (20 + 40)}{2} + 2 \cdot 800 + \frac{47.1^2}{4 \cdot 800} \cdot (cosec\frac{180}{20} - cosec\frac{180}{40})^2$$

$$L = 1413 + 1600 + 27.98 = 3040.9\ mm$$