Coulomb's Law and Electric Field Intensity

Coulomb stated that the force between two very small objects separated in a vacuum or free space by a distance, which is large compared to their size, is proportional to the charge on each and inversely proportional to the square of the distance between them.

$$F = k\frac{Q_1 Q_2}{R^2} \qquad \text{(Newton)}$$

where $Q1$ and $Q2$ are the positive or negative quantities of charge in coulomb (C) , R is the separation between the charges in meter (m), F is the force in newton (N), If the International System of Units1 (SI) is used and k is a proportionality constant:

$$k = \frac{1}{4\pi\epsilon_0}$$

ϵ_0 : is the ***permittivity* (السماحية الكهربائية في الفراغ)** *of free space* and has magnitude, measured in farads per meter (F/m).

$$\epsilon_\circ = 8.854 \times 10^{-12} = \frac{10^{-9}}{36\pi} \qquad (F/m)$$

Coulomb's law is now :

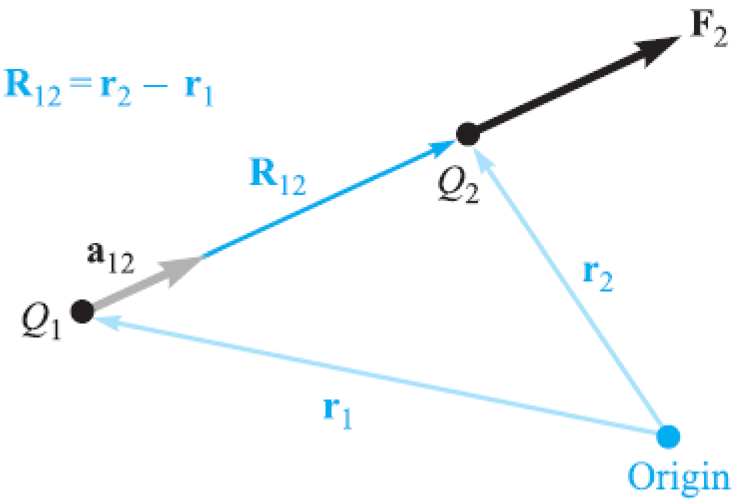
$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \qquad \text{(N)}$$

In vector form:

$$\mathbf{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}\,\mathbf{a} \qquad \text{(N)}$$ قانون كولوم بالصيغة الأتجاهية

a: auint vector in the direction of the force.

Let the vector **r**1 locate *Q*1, whereas **r**2 locates *Q*2. Then the vector
R12 = **r**2 − **r**1
represents the directed line segment from *Q*1 to *Q*2, as shown in Figure 2.1.

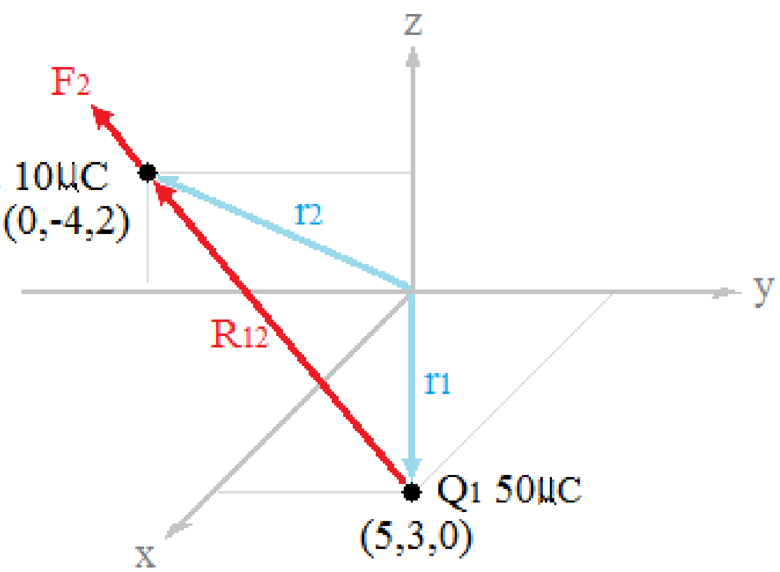


the unit vector **a12** is:

$$\mathbf{a}_{12} = \frac{\boldsymbol{R}_{12}}{|\boldsymbol{R}_{12}|} = \frac{\mathbf{r2} - \mathbf{r1}}{|\boldsymbol{r2} - \boldsymbol{r1}|}$$

Example: Find the force **F** act on Q2 (10μc at (0,-4,2)) due to Q1(50 μc at (5,3,0)) in free space, see figure (1).

Solution:



Figure(1)

R_{12}=(0-5)**ax**+(-4-3)**ay**+(2-0)**az** = -5**ax**-7**ay**+2**az**

$$\mathbf{a_{12}} = \frac{\boldsymbol{R_{12}}}{|\boldsymbol{R_{12}}|} = \frac{\mathbf{-5ax-7ay+2az}}{\sqrt{-5^2+7^2+2^2}} = \frac{\mathbf{-5ax-7ay+2az}}{\sqrt{78}}$$

$R_{12}=\sqrt{78}$ (المسافة بين الشحنتين)

$$\mathbf{F_2} = \frac{Q_1Q_2}{4\pi\epsilon_0 R_{12}^2}\ a_{12}$$

$$\mathbf{F_2} = \frac{50\times10^{-6}\times10\times10^{-6}}{4\pi\frac{10^{-9}}{36\pi}(78)}\ \frac{-5\mathbf{ax}-7\mathbf{ay}+2\mathbf{az}}{\sqrt{78}}$$

F_2=(-32.66 **ax** – 45.72 **ay** + 13.06 **az**) mN

The force act on Q1 by Q2 is **F1**= - **F2**

F_1=(32.66 **ax** + 45.72 **ay** - 13.06 **az**) mN

$$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{Q_1Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{21} = -\frac{Q_1Q_2}{4\pi\epsilon_0 R_{12}^2}\mathbf{a}_{12}$$

D2.1. A charge $Q_A = -20\,\mu$C is located at $A(-6, 4, 7)$, and a charge $Q_B = 50\,\mu$C is at $B(5, 8, -2)$ in free space. If distances are given in meters, find: (*a*) $\mathbf{R}_{AB}$; (*b*) R_{AB}. Determine the vector force exerted on Q_A by Q_B if $\epsilon_0 =$ (*c*) $10^{-9}/(36\pi)$ F/m; (*d*) 8.854×10^{-12} F/m.

Ans. $11\mathbf{a}_x + 4\mathbf{a}_y - 9\mathbf{a}_z$ m; 14.76 m; $30.76\mathbf{a}_x + 11.184\mathbf{a}_y - 25.16\mathbf{a}_z$ mN; $30.72\mathbf{a}_x + 11.169\mathbf{a}_y - 25.13\mathbf{a}_z$ mN