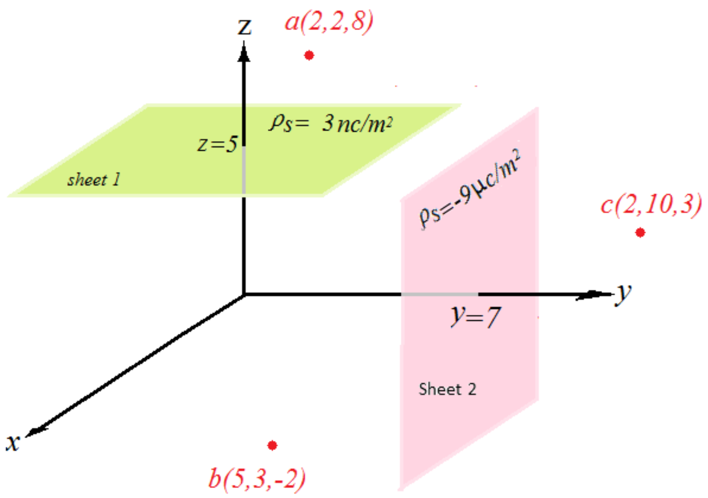
Example1:Two infinit uniform sheets of charge in free space. Sheet1 of ρs=3nc/m² is located at z=5, sheet 2 of ρs=-9μc/m² is located at y=7. Find the electric field intensity at the following points:
a(2,2,8), b(5,3,-2), c(2,10,3).



Solution:

$$E = \frac{\rho_s}{2\epsilon_0} a_N$$

point (a): $E1 = \frac{3\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}(az)$ $\quad ,E2 = \frac{-9\times10^{-6}}{2\times\frac{10^{-9}}{36\pi}}(-ay)$

Ea= E1+E2 $\qquad Ea = \frac{3\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}(az) + \frac{-9\times10^{-6}}{2\times\frac{10^{-9}}{36\pi}}(-ay)$

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Ea= $508.94 \times 10^3 \boldsymbol{ay} + 169.65\ \boldsymbol{az}$ v/m

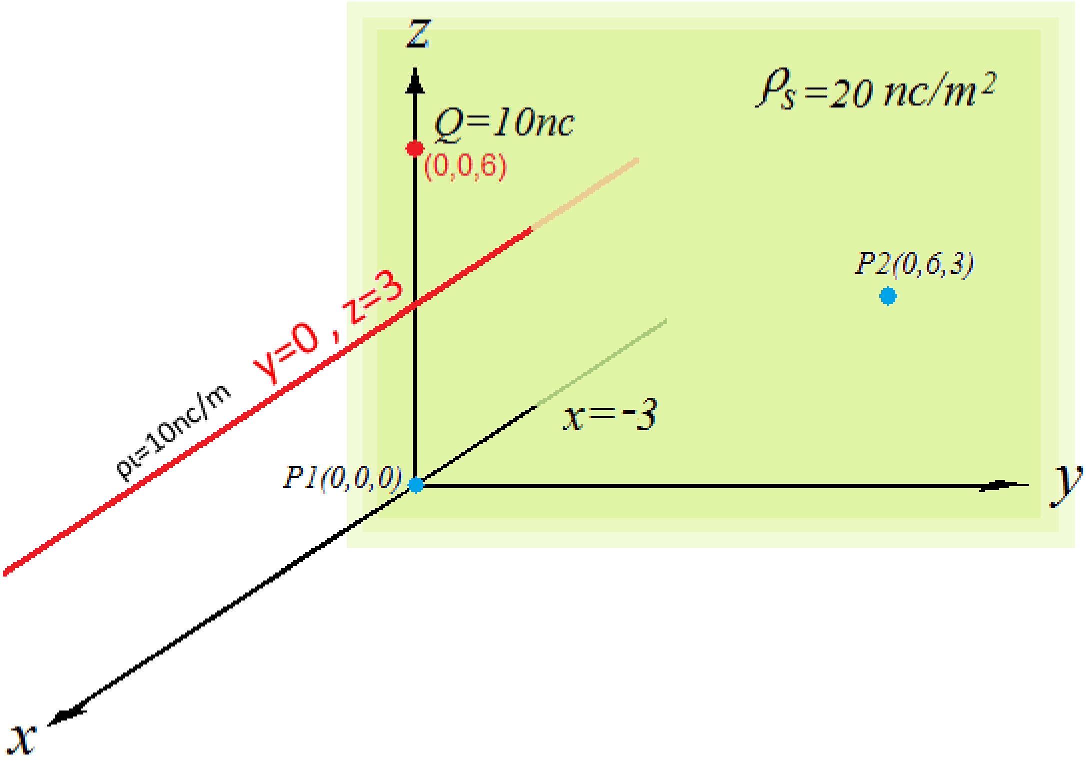
Eb= $508.94 \times 10^3 \boldsymbol{ay} - 169.65\ \boldsymbol{az}$ v/m

Ec= $-508.94 \times 10^3 \boldsymbol{ay} - 169.65\ \boldsymbol{az}$ v/m

Example2: In free space, a point charge of 20 nc at (0,0,6) , an infinit line charge of ρ_L=10 nc/m at y=0, z=3, and an infinit sheet of charge of ρ_S=20 nc/m² at x= -3.

Find the electric field intensity at: i) P1(0,0,0) , ii) P2(0,6,3).

Solution: As shown in figure below:



The field at the point P1 or P2 is equal to the sum of fields of all charge types:

E(total) **=** **E**(point charge)+**E**(line) +**E**(sheet)

i) P(0,0,0)

$$\mathbf{E}_Q = \frac{Q}{4\pi\epsilon_o R^2}\,\boldsymbol{a_r}$$

$$\boldsymbol{a_r} = \frac{(0-0)\boldsymbol{ax}+(0-0)\boldsymbol{ay}+(0-6)\boldsymbol{az}}{\sqrt{0^2+0^2+(-6)^2}} = \frac{-6\boldsymbol{az}}{6} = -\boldsymbol{az}$$

$$\mathbf{E}_Q = \frac{10\times10^{-9}}{4\pi\frac{10^{-9}}{36\pi}36}(-\mathbf{az}) = -2\cdot5\,\boldsymbol{az} \quad v/m$$

$$\boldsymbol{E_L} = \frac{\rho_l}{2\pi\epsilon_o\rho}\boldsymbol{a}_\rho = \frac{10\times10^{-9}}{2\pi\frac{10^{-9}}{36\pi}3}\ \frac{-3\boldsymbol{a}z}{3} = -\ 60\ \mathbf{az}\ \text{v/m}$$

$$\boldsymbol{E}_s = \frac{\rho_s}{2\epsilon_o}a_N = \frac{20\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}(\boldsymbol{ax}) = 360\pi\ \boldsymbol{ax} = 1130.97\boldsymbol{ax}\quad v/m$$

$$\boldsymbol{E}_{(0.0.0)} = \boldsymbol{E}_Q = \boldsymbol{E}_l + \boldsymbol{E}_s$$

$$\boldsymbol{E}_{(0.0.0)} = -2.5\ \boldsymbol{az} - 60\boldsymbol{az} + 1130.97\boldsymbol{ax}$$

$$\boldsymbol{E}_{(0.0.0)} = 1130.97\boldsymbol{ax} - 62.5\boldsymbol{az}\quad v/m$$

ii) P(0,6,3)

$$\mathbf{E}\text{Q}=\frac{Q}{4\pi\epsilon_o R^2}\boldsymbol{a_r}$$

$$\boldsymbol{a}_r = \frac{(0-0)\boldsymbol{ax}+(6-0)\boldsymbol{ay}+(3-6)\boldsymbol{az}}{\sqrt{0^2+6^2+(-3)^2}} = \frac{6\boldsymbol{ay}-3\boldsymbol{az}}{\sqrt{45}}$$

$$\mathbf{E}\text{Q}=\frac{10\times10^{-9}}{4\pi\frac{10^{-9}}{36\pi}45}\ \frac{6\boldsymbol{ay}-3\boldsymbol{az}}{\sqrt{45}} = 2\,\frac{6\boldsymbol{ay}-3\boldsymbol{az}}{\sqrt{45}} = 1.79\boldsymbol{ay} - 0.89\boldsymbol{az}\quad v/m$$

$$\boldsymbol{E_L} = \frac{\rho_l}{2\pi\epsilon_o\rho}\boldsymbol{a}_\rho = \frac{10\times10^{-9}}{2\pi\frac{10^{-9}}{36\pi}6}\ \frac{6\boldsymbol{ay}}{6} = 30\ \mathbf{ay}\ v/m$$

$$\boldsymbol{E}_s = \frac{\rho_s}{2\epsilon_o}\boldsymbol{a_N} = \frac{20\times10^{-9}}{2\times\frac{10^{-9}}{36\pi}}(\boldsymbol{ax}) = 360\pi\ \boldsymbol{ax} = 1130.97\boldsymbol{ax}\quad v/m$$

$$\boldsymbol{E}_{(0.6.3)} = \boldsymbol{E}_Q = \boldsymbol{E}_l + \boldsymbol{E}_s$$

$$\boldsymbol{E}_{(0.6.3)} = (1.79\boldsymbol{ay} - 0.89\boldsymbol{az}) + (30\,\mathbf{ay}) + (1130.97\boldsymbol{ax})$$

$$\boldsymbol{E}_{(0.6.3)} = 1130.97\boldsymbol{ax} + 31.79\boldsymbol{ay} - 0.89\boldsymbol{az} \quad v/m$$