

هذان سؤالان محلولان عن موضوع المتجهات, حاول ان تحل بنفسك بعد مراجعة المحاضرة والاطلاع على المصادر ثم قارن حلك مع الحل الموجود.

Q1) In Cartesian coordinate system, a point P1(-1,-4,0), and point

P2(2,-2,1). Find the following :

1. A position vector **r1**.
2. A position vector **r2**.
3. A displacement vector **r12**.
4. A displacement vector **r21**.
5. $|\mathbf{r1}|$.
6. $|\mathbf{r2}|$.
7. **ar1**
8. **ar2**
9. **ar21**.
10. **r1 + r2**
11. **r1 - r2**
12. The components of **r12**
13. The magnitudes of components of **r12**
14. **4r12**
15. **r21/-3**

Solution

1. $\mathbf{r1} = -\mathbf{ax} - 4\mathbf{ay}$
2. $\mathbf{r2} = 2\mathbf{ax} - 2\mathbf{ay} + \mathbf{az}$
3. $\mathbf{r12} = (2 - (-1))\mathbf{ax} + (-2 - (-4))\mathbf{ay} + (1 - 0)\mathbf{az} = 3\mathbf{ax} + 2\mathbf{ay} + \mathbf{az}$
4. $\mathbf{r21} = -3\mathbf{ax} - 2\mathbf{ay} - \mathbf{az}$
5. $|\mathbf{r1}| = \sqrt{(-1)^2 + (-4)^2 + (0)^2} = \sqrt{17} = 4.123$
6. $|\mathbf{r2}| = \sqrt{(2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$
7. $\mathbf{ar1} = \frac{\mathbf{r1}}{|\mathbf{r1}|} = \frac{-\mathbf{ax} - 4\mathbf{ay}}{\sqrt{17}} = -\frac{1}{\sqrt{17}}\mathbf{ax} - \frac{4}{\sqrt{17}}\mathbf{ay}$
8. $\mathbf{ar2} = \frac{\mathbf{r2}}{|\mathbf{r2}|} = \frac{2\mathbf{ax} - 2\mathbf{ay} + \mathbf{az}}{3} = \frac{2}{3}\mathbf{ax} - \frac{2}{3}\mathbf{ay} + \frac{1}{3}\mathbf{az}$

$$9. \mathbf{ar21} = \frac{\mathbf{r21}}{|\mathbf{r21}|} = \frac{-3\mathbf{ax} - 2\mathbf{ay} - \mathbf{az}}{\sqrt{9+4+1}} = \frac{-3}{\sqrt{14}}\mathbf{ax} - \frac{2}{\sqrt{14}}\mathbf{ay} - \frac{1}{\sqrt{14}}\mathbf{az}$$

$$10. \mathbf{r1+r2} = (-1+2)\mathbf{ax} + (-4-2)\mathbf{ay} + (0+1)\mathbf{az} = 2\mathbf{ax} - 6\mathbf{ay} + \mathbf{az}$$

$$11. \mathbf{r1-r2} = \mathbf{r21} = -3\mathbf{ax} - 2\mathbf{ay} - \mathbf{az}$$

12. The components of $\mathbf{r12}$ is a three vectors as follows

1) $3\mathbf{ax}$ (in x direction)

2) $2\mathbf{ay}$ (in y direction)

3) \mathbf{az} (in z direction)

13. The matnituds of components of $\mathbf{r12}$ is

1) $A_x=3$

2) $A_y=2$

3) $A_z=1$

$$14. 4\mathbf{r12}=4(3\mathbf{ax}+2\mathbf{ay}+\mathbf{az}) = 12\mathbf{ax}+8\mathbf{ay}+4\mathbf{az}$$

$$15. \frac{\mathbf{r21}}{-3} = \frac{1}{-3} (-3\mathbf{ax} - 2\mathbf{ay} - \mathbf{az}) = \mathbf{ax} + \frac{2}{3}\mathbf{ay} + \frac{1}{3}\mathbf{az}$$

Q2)

Find the angle between two vectors $\mathbf{A}=3\mathbf{ax} + 4\mathbf{ay} - \mathbf{az}$ and $\mathbf{B}= 2\mathbf{ax} - \mathbf{ay} + \mathbf{az}$.

solution:

$$\mathbf{A} \cdot \mathbf{B} = (3\mathbf{ax} + 4\mathbf{ay} - \mathbf{az}) \cdot (2\mathbf{ax} - \mathbf{ay} + \mathbf{az})$$

$$= (3)(2) + (4)(-1) + (-1)(1)$$

$$= 6-4-1$$

$$= 1$$

Thus, $\mathbf{A} \cdot \mathbf{B} = 1$

The Magnitude of vectors is given by

$$|\mathbf{A}| = \sqrt{(3)^2 + (4)^2 + (-1)^2} = \sqrt{26} = 5.1$$

$$|\mathbf{B}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6} = 2.45$$

By the form:

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta_{AB}$$

$$\cos(\theta_{AB}) = \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}| |\mathbf{B}|}$$

The angle between the two vectors is

$$\theta_{AB} = \cos^{-1} \left[\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}| |\mathbf{B}|} \right]$$

$$\theta = \cos^{-1} = 1/(5.1)(2.45)$$

$$\theta = \cos^{-1} = (0.0802)$$

$$\theta = 85.41^\circ$$